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MATHEMATICS REVIEW SUPPLEMENT (for Preparation for Grade Nine)

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MATHEMATICS REVIEW SUPPLEMENT (for Preparation for Grade Nine)

Mathematics 9
Review Supplement
Sections 1-8
Alberta Distance Learning Centre
ISBN No. 0-7741-1150-X

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MATHEMATICS
REVIEW SUPPLEMENT
(for Preparation for Grade Nine)

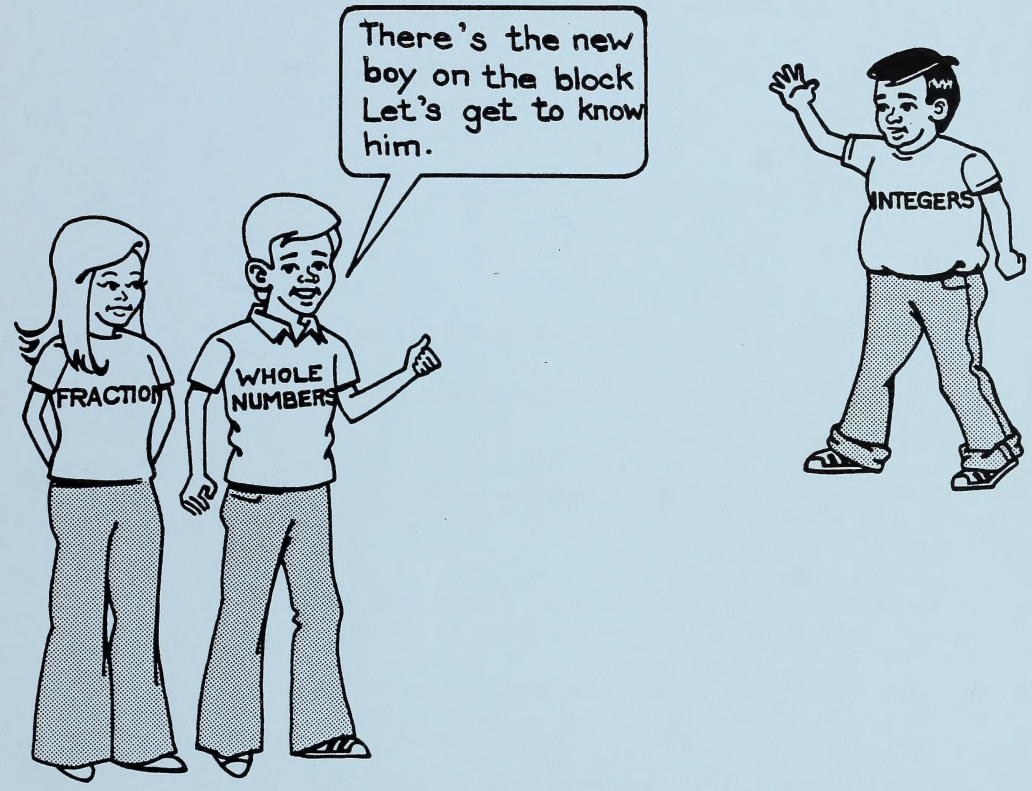
This supplement is a review of grade 8 MATHEMATICS. It is intended to help you with your Math 9 course. We strongly recommend that you work through all of this review supplement before you start your Math 9 course.

Answers for the exercises are found at the end of each section.

Sections 1 through 5 reviews Integers.
Sections 6 and 7 reviews Fractions.
Section 8 reviews Algebra-Solving Equations.

Teaching Assistance: If you require help with a course, you can telephone toll-free to the School in Barrhead and ask for the appropriate department. Teachers are available to provide assistance to students during the entire year including July and August. Please dial your local RITE number and ask for the Alberta Distance Learning Centre. The RITE number is usually the first number listed under Government of Alberta. In the Edmonton exchange area, you may dial 427-2711 and ask for the Alberta Distance Learning Centre. In areas of Alberta not served by the RITE system, dial 0 and ask for Zenith 22333, then request the Alberta Distance Learning Centre. Calls may be made from 8:15 a.m. to 4:30 p.m. Monday to Friday.

INTEGERS



In this section you will learn about:

- the meaning of integers
- the order of integers
- absolute value

UNDERSTANDING INTEGERS

Number Sets

In your study of numbers so far, you have learned about:

(i) the set of whole numbers,

$\{ 0, 1, 2, 3, \ldots \}$

(ii) the set of common fractions,

$\{ \frac{a}{b}$ where a and b are whole numbers and $b \neq 0 \}$

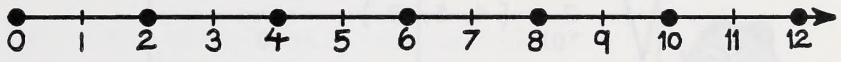
(iii) the set of decimals - so far just another way of writing fractions.

You also learned how to plot these numbers on a number line.

Examples

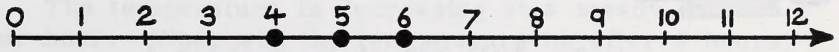
1. Plot the even whole numbers on a number line.

The even whole numbers are $\{ 0, 2, 4, 6, 8, \ldots \}$.

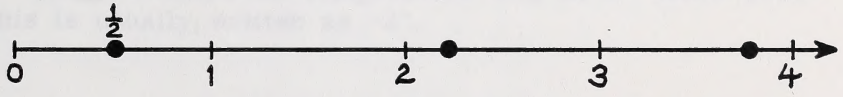


Note: The set is plotted right up to the arrow to show that is is an infinite set (goes on forever).

2. Plot the whole numbers between 3 and 7 on a number line. The numbers are $\{ 4, 5, 6 \}$.



3. Plot $\{ \frac{1}{2}, 2\frac{1}{4}, 3.81 \}$ on a number line.



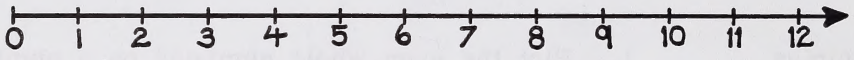
(The position of these points has been estimated.)

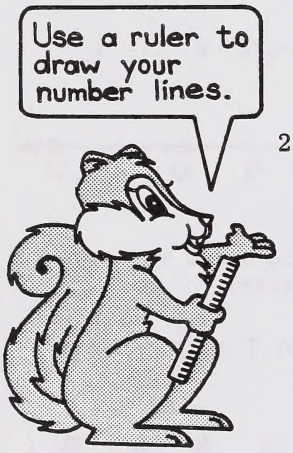
Remember:

- the whole number line starts at 0 and goes to infinity,
- the whole numbers must be evenly spaced on the number line,
- the numbers that are to be plotted are shown by circling them or marking them with a large dot on the number line.

EXERCISE 1 Plot each of the following sets on a number line.

1. { 1, 4, 7, 10, 13, ... }





2. { 4, 5, 8 }

3. { all odd whole numbers }

Need for New Numbers

The set of fractions was invented to solve equations such as:

$$4x = 3$$

Using fractions, the solution becomes:

$$x = 3 \div 4$$

$$x = \frac{3}{4}$$

However, equations such as $x + 6 = 3$ have no solution in the set of whole numbers or the set of fractions.

As well, numbers are needed to express oppositeness:

an asset of \$3 as opposed to a deficit of \$3

5 km east as opposed to 5 km west

Example

Consider a scientist doing an experiment in a laboratory. He carefully records the temperature of a substance he is studying. Here is part of his table of time and temperature.

Hours Elapsed	Temperature
0	10°
1	8°
2	6°
3	4°

The temperature is decreasing at a steady rate of 2° per hour. What will the temperature be after 4 hours, after 5 hours, after 6 hours?

After 4 hours the temperature will be $4 - 2 = 2°$. After 5 hours it will be 0°.

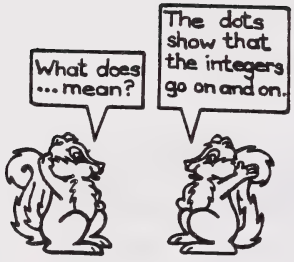
After 6 hours the temperature will be 2° below zero. This is usually written as -2°.

And so, the scientist's table would look like this.

Hours Elapsed	Temperature
0	10°
1	8°
2	6°
3	4°
4	2°
5	0°
6	-2°
7	-4°

The numbers used in this table are called integers.

The set of integers is written as:



$I = \{. \ . \ . \ -4, -3, -2, -1, 0, +1, +2, +3, \ . \ . \ .\}$

Integers less than zero are called negative integers and integers greater than zero are called positive integers. Zero is still called zero and has no sign. There are no fractions in the set of integers.

Collectively, the elements of I are sometimes referred to as the set of positive and negative whole numbers and zero.

The plus and minus signs are used to show oppositeness.

Examples

1. The temperature went up by 10°C. +10
2. The stock market made an 8 point gain. +8
3. George spent $5.00. -5
4. The car slowed down by 20 km/h. -20
5. The elevator went up 7 floors. +7
6. The birds flew 2000 km south. -2000
   (Note: South is often considered a negative direction and north is positive.)

EXERCISE 2 Represent each of the following expressions with an integer. Some are done for you.

1. 10° below zero _____
2. 50 years hence +50
3. 5 m below street level _____
4. 38° N latitude _____
5. a balance of $150 _____
6. assets of $10 895 _____
7. a debt of $109 −$109
8. a loss of $50 _____
9. $10 saved _____
10. income of $5000 _____
11. a score of 500 _____
12. a bonus of 10 marks _____
13. 30° above zero _____
14. 35 years ago _____
15. $10 spent _____

16. Explain the net change in each of the following, using first a sentence and then an integer.

(a) An advance of 6 km and a withdrawal of 3 km.

The result is an overall advance of 3 km.

+3

(b) An elevator goes up 3 floors and then goes down 5 floors.

_____

_____

(c) A stock drop of 3 points and then a rise of 5 points.

_____

_____

(d) A ship steams 50 km east and then 40 km west.
Hint: Let east be the positive direction.

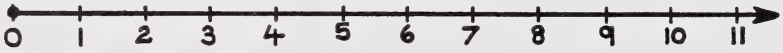
_____

_____

NUMBER LINES

Whole Numbers

You have already had experience in graphing whole numbers on a number line. It looked like this.

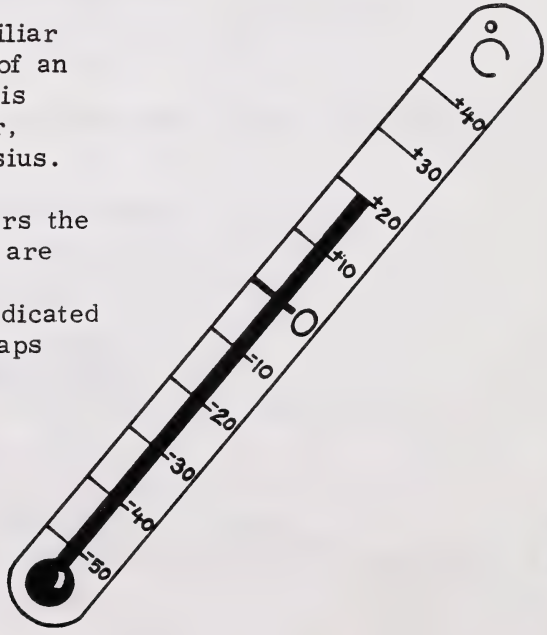


The dot on the left end means that the numbers start at 0. The arrow on the right end means that the numbers go on forever in that direction.

Integers

You are already familiar with a common example of an integer number line. It is the ordinary thermometer, either Fahrenheit or Celsius.

On some thermometers the temperatures below zero are not written as negative numbers, but they are indicated in some other way, perhaps a different colour.



Here is the set of integers written out again.

$I = \{ \ldots -3, -2, -1, 0, +1, +2, +3, \ldots \}$

They are represented on a number line as follows.

This time there are arrows on both ends. The integer line goes on forever in both directions.

If you cover up all the parts of the line to the left of 0, you are left with the whole number line.

It is not necessary to put the + signs on the positive integers. You did not use them when you called the positive integers whole numbers. The + signs have been used here to emphasize the difference between positive and negative integers.

It is, however, absolutely necessary to use the - sign for negative integers.

EXERCISE 3

1. How does the integer number line differ from the whole number line?

_____

_____

_____

2. What do the arrows indicate at both the left and right of the integer number line?

_____

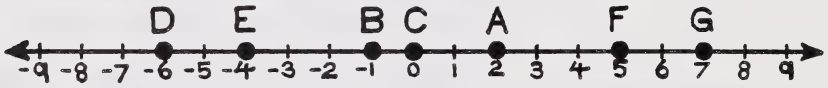
_____

_____

2. Draw an integer number line like the one on page 8 and mark the following points on it. Use large dots or circles to graph the numbers, just as you did for whole numbers. Use a ruler to draw your number line.

(a) 4	(b) -2	(c) 0
(d) -1	(e) -5	(f) 3

3. Which integer is represented by each of the following letters on the number line below.



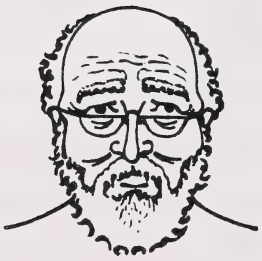
A = _____ B = _____ C = _____ D = _____

E = _____ F = _____ G = _____

Inequality Signs

In previous work, you have probably used the signs <, =, and >. Of course, = is an equals sign. < and > are called inequality signs.

< means "is less than" for example, 3 < 5
> means "is greater than" for example, 10 > 6



GEORGE'S AGE > TIMMY'S AGE

GEORGE

TIMMY

Order of Integers

To order a set of numbers means to put them in order of size. You already know how to order the whole numbers.

$$\{0, 1, 2, 3, 4, 5, \ldots\}$$

The farther you move to the right, the larger the number.

The farther you move to the left, the smaller the number.

Zero is the smallest whole number. There is no largest whole number.

The same ordering principle applies to the integers.

$$\{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

The farther you move to the right, the larger the number.

The farther you move to the left, the smaller the number.

There is no smallest integer. There is no largest integer.

This does not sound too difficult, but it can sometimes be confusing.

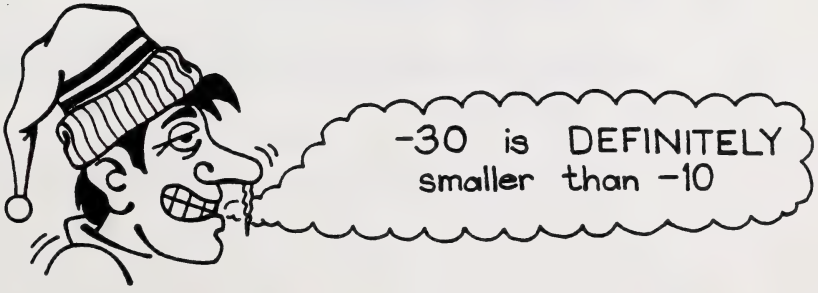
Example

4 is greater than 2, but -4 is smaller than -2.

The negative integers, are opposite in order to the positive integers.

If you are ever in doubt, think of the thermometer. Instead of thinking about which number is greater, think about which number represents a higher (warmer) temperature.

Conversely, instead of thinking about which number is smaller, think about which number represents a lower (colder) temperature.



EXERCISE 4

1. Write the opposite of each of the following integers.

(a) -6 +6 (b) +3 -3

(c) -5 _____ (d) -9 _____

(e) +18 _____ (f) -100 _____

2. Write two integers which fit each of the following descriptions.

(a) less than +6 _____ _____

(b) greater than +3 _____ _____

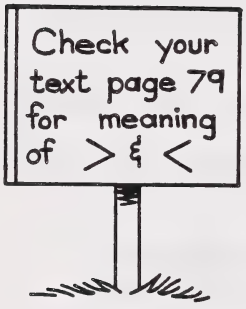
(c) less than -2 _____ _____

(d) greater than -4 _____ _____

(e) greater than -1 but less than +6 _____ _____

(f) less than -2 but greater than -8 _____ _____

3. Write < or > in the blank to make each of the following a true statement.



(a) 4 _____ 8 (b) -3 _____ -2

(c) +1 _____ -1 (d) 0 _____ -4

(e) -80 _____ -20 (f) 0 _____ +1

(g) 6 _____ -8 (h) +3 _____ +5

4. Write these integers in order, from least to greatest.

(a) -2, +3, -4, +2, -8, +1

_____

(b) -2, +3, -4, +5, -6, +7, -8

_____

(c) +1, -1, +5, -5

_____

5. Choose an integer. Then name the next greatest integer, that is, its **successor.** For example: -5 is the successor of -6.

   (a) Integer _____ Successor _____

   (b) How do you find the successor of an integer? _____

   _____

   (c) Does every integer have a successor? _____

6. Choose an integer. Then name the next smaller integer, that is, its **predecessor.** For example: -7 is the predecessor of -6.

   (a) Integer _____ Predecessor _____

   (b) How do you find the predecessor of an integer? _____

   _____

   (c) Does every integer have a predecessor? _____

7. (a) Is there a greatest integer? _____

   Explain _____

   _____

   (b) Is there a least integer? _____

   Explain _____

   _____

   (c) What is the greatest whole number? _____

   (d) What is the least whole number? _____

8. Write the set of integers for each of the following descriptions. Use the proper set notation.

   (a) all integers greater than -2 _____

   (b) all integers less than 3 _____

GRAPHING INEQUALITIES

Reference

In order to graph an equality, a set of points must be graphed on the integer number line. For example, consider $n > -4$ if $n \in I$.

The solution set will be all the integers which lie to right of -4 on the integer line. Why? ....

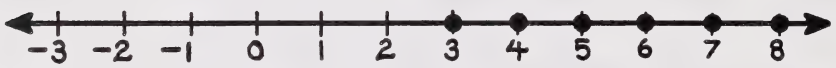
The solution set is {-3, -2, -1, 0, +1, +2, ...}

Example 1

Graph the solution set of $n \geqq 3$, $n \in I$

The difference between $n > 3$ and $n \geqq 3$ is that the number 3 is included in $n \geqq 3$.

The solution set of $n \geqq 3$ is {3, 4, 5, ...}.



Note that all the numbers right up to the arrow are marked to indicate that the set goes on forever in that direction. This is an infinite set.

Example 2

Graph the solution set of $-4 < x$, $x \in I$.

The important thing to notice here is that:

$-4 < x$ means the same as $x > -4$.

That is, -4 is less than x means the same as x is greater than -4.

The solution set is {-3, -2, -1, 0, 1, 2, ...}

Note: (a) -4 is not included.

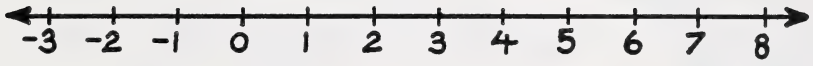
(b) All numbers are marked right up to the arrow to show that the set goes on forever (is an infinite set.)

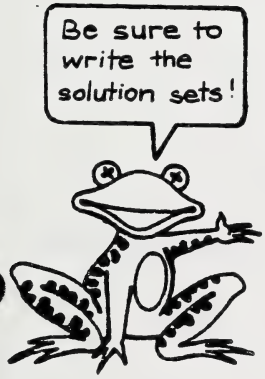
$\in$ $\in$ means "is a member of" or "belongs to" so that $x \in I$ means that x is a member of the set of Integers.

SELF-CORRECTING EXERCISE A

For each of the following inequalities write the solution set. Then, graph it on a number line.

1. $x \geqq 5,\ x \in I$





2. $y < -3,\ y \in I$

3. $-2 < w,\ w \in I$

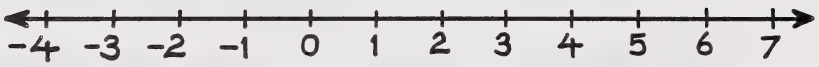
4. $-1 \leqq k,\ k \in I$

Check your work with the answers on page 20 of this section. Be sure to correct your errors.

EXERCISE 5

1. For each of the following inequalities, write the solution set and graph it on a number line.

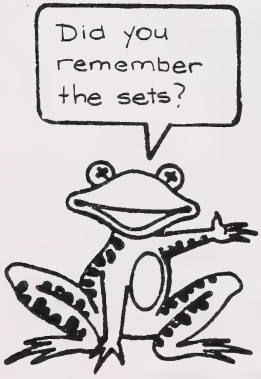
(a) $h < 5,\ h \in I$



(b) $n \geqq 1,\ n \in I$

(c) $k \leqq -3,\ k \in I$

(d) $-7 \leqq c,\ c \in I$



(e) $-8 < k,\ k \in I$

EXERCISE 5 (continued)

2. Complete the following sentences using the words "greater" or "less" to make the sentence true.

   (a) Zero is _____ than all the positive integers.

   (b) A negative number is always _____ than zero.

   (c) A positive number is always _____ than a negative number.

   (d) A negative number is always _____ than a positive number.

   (e) A positive number is always _____ than zero.

   (f) Zero is _____ than all the negative numbers.

ABSOLUTE VALUE

Integers are numbers that have both magnitude and direction. These numbers are very useful and necessary in the study of Mathematics.

The direction is indicated by the + or - sign.

The magnitude is indicated by the numeral without the sign.

In mathematics, the magnitude of a number is called its absolute value. This is a positive quantity. It is indicated by two vertical lines.

$|-6|$ means "the absolute value of -6".

$|+4|$ means "the absolute value of +4".

A good way to find the absolute value of a number is to look at how many spaces away from zero it is on the number line.

Examples $|-6| = 6$ since -6 is 6 spaces away from zero.

$|+4| = 4$ since +4 is 4 spaces away from zero.

Here is the formal definition.

Absolute Value

> If a is any whole number, except zero
>
> $|+a| = a$ (The absolute value of +a is a.)
>
> $|-a| = a$ (The absolute value of -a is a.)
>
> The absolute value of zero is zero.

The last part of the definition may seem trivial but it is necessary to state it.

$|0| = 0$ since zero is 0 spaces away from zero.

Example 1 Find the absolute value of -2, +101, -7, and +45.

$|-2| = 2$ $|-7| = 7$

$|+101| = 101$ $|+45| = 45$

Example 2 What integer has the same absolute value as -5?

$|-5| = 5$

Think: What other integer is 5 spaces from zero on the number line?

$|+5| = 5$

Thus, -5 and +5 have the same absolute value.

Example 3 Which integer has the greater absolute value, -10 or +3?

$|-10| = 10$ $|+3| = 3$

$10 > 3$

Thus, -10 has a greater absolute value than +3.

EXERCISE 6

1. Find the following absolute values. Use a number line, if necessary.

(a) $|+7| =$ (b) $|-4| =$

(c) $|-126| =$ (d) $|+85| =$

(e) $|0| =$ (f) $|-1| =$

2. For each of the following numbers, write another integer which has the same absolute value.

(a) +6 _____ (b) -4 _____

(c) -81 _____ (d) +10 _____

(e) +1 _____ (f) 0 _____

3. In each of the following pairs of numbers, pick out the number with the greater absolute value. Follow example 3, on page 17.

(a) -6, +4

$|-6| = 6$ $|+4| = 4$

$6 > 4$

-6 has the greater absolute value.

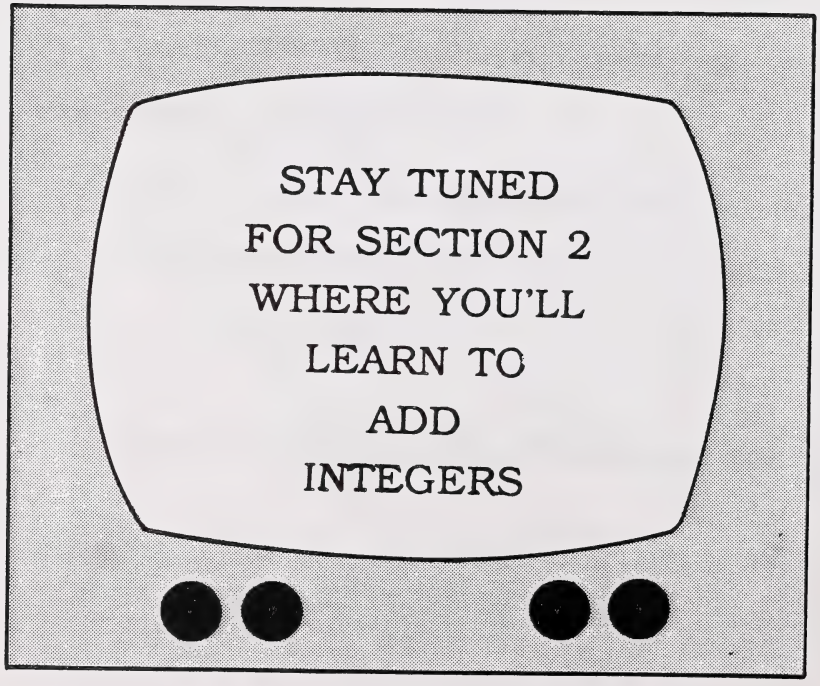
(b) -14, +10

(c) +25, -8

(d) +6, +10

(e) +168, -150

(f) -16, +16



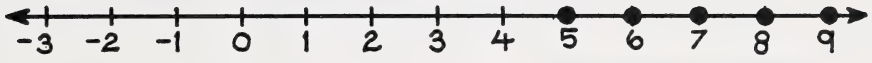
KEY TO SELF-CORRECTING EXERCISES IN SECTION 1

SELF-CORRECTING EXERCISE A, page 14.

Did you use a ruler for your number lines and did you space the numbers evenly?

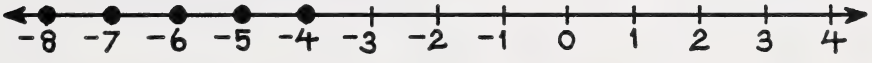
1. $x \geqq 5,\ x \in I$

   Solution Set = { 5, 6, 7, 8, ... }



2. $y < -3,\ y \in I$

   Solution Set = { ... -6, -5, -4 }

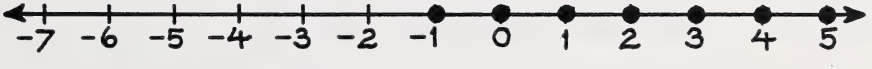


3. $-2 < w,\ w \in I$

   Solution Set = { -1, 0, 1, 2, ... }

4. $-1 \leqq k,\ k \in I$

   Solution Set = { -1, 0, 1, 2, ... }

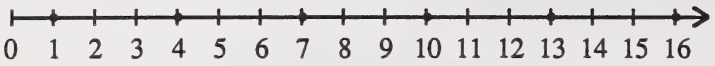


Note: This answer is the same as the answer for Question 3.

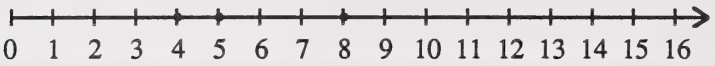
Key to the Exercises in Section 1

Exercise 1

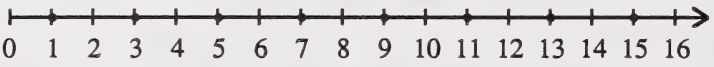
1. {1, 4, 7, 10, 13,. . .}



2. {4, 5, 8}



3. {all odd whole numbers}



Exercise 2

1. −10
2. +50
3. −5
4. +38
5. +150
6. +10 895
7. −$109
8. −50
9. +10
10. +5000
11. +500
12. +10
13. +30
14. −35
15. −10
16. (a) The result is an overall advance of 3 km: +3

    (b) The result is 2 floors below the starting point: −2

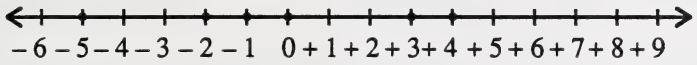
    (c) The result is an overall rise of 2 points: +2

    (d) The result is 10 km east: +10

Exercise 3

1. It now includes the negative numbers.

2. The number line continues in both directions. There is no end to integers.

3. 

4. $A = 2$ $B = -1$ $C = 0$ $D = -6$
$E = -4$ $F = 5$ $G = 7$

Exercise 4

1. (a) $+6$ (b) -3
(c) $+5$ (d) $+9$
(e) -18 (f) $+100$

2. (a) $\{5, 4, 3, 2, \ldots\}$
(b) $\{4, 5, 6, 7, \ldots\}$
(c) $\{-3, -4, -5, \ldots\}$
(d) $\{-3, -2, -1, \ldots\}$
(e) $\{0, 1, 2, 3, 4, 5\}$
(f) $\{-3, -4, -5, -6, -7. \ldots\}$

3. (a) $4 < 8$ (b) $-3 < -2$

   (c) $+1 > -1$ (d) $0 > -4$

   (e) $-80 < -20$ (f) $0 < +1$

   (g) $6 > -8$ (h) $+3 < +5$

4. (a) $-8, -4, -2, +1, +2, +3$

   (b) $-8, -6, -4, -2, +3, +5, +7$

   (c) $-5, -1, +1, +5$

5. (a) Integer -1 Successor 0 (many possible answers)

   (b) Add one to your integer.

   (c) Yes

6. (a) Integer -1 Predecessor -2 (many possible answers)

   (b) Subtract one from your integer.

   (c) Yes

7. (a) No. Integers have no limits

   (b) No. No limit to integers.

   (c) None

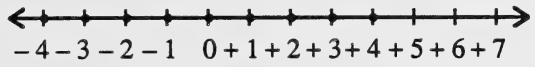
   (d) 0

8. (a) $\{-1, 0, 1, 2, \ldots\}$

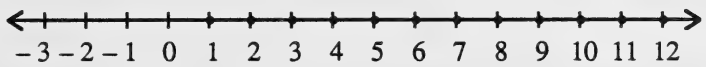
   (b) $\{\ldots, -1, 0, 1, 2\}$

Exercise 5

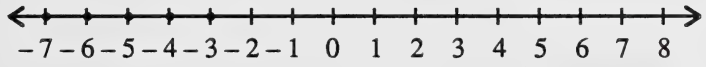
1. (a) $h < 5$, $h \in I$ Solution Set $\{\ldots -1, 0, 1, 2, 3, 4\}$



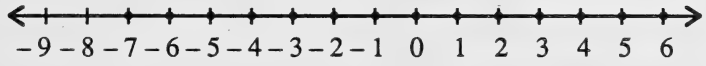
   (b) $n \geq 1$, $n \in I$ Solution Set $\{1, 2, 3, \ldots\}$



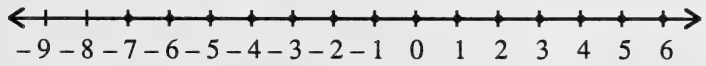
   (c) $k \leq -3$, $k \in I$ Solution Set $\{\ldots -6, -5, -4, -3\}$



   (d) $-7 \leq c$, $c \in I$ Solution Set $\{-7, -6, -5, -4, \ldots\}$



   (e) $-8 < k$, $k \in I$ Solution Set $\{-7, -6, -5, \ldots\}$



2. (a) less

   (b) less

   (c) greater

   (d) less

   (e) greater

   (f) greater

Exercise 6

1. (a) $|+7| = 7$
   (b) $|-4| = 4$
   (c) $|-126| = 126$
   (d) $|+85| = 85$
   (e) $|0| = 0$
   (f) $|-1| = 1$
2. (a) $+6,\ -6$
   (b) $-4,\ +4$
   (c) $-81,\ +81$
   (d) $+10,\ -10$
   (e) $+1,\ -1$
   (f) $0,\ 0$
3. (a) $|-6| = 6$ $\quad |+4| = 4$

   $6 > 4$; -6 has the greater absolute value

   (b) $|-14| = 14$ $\quad |+10| = 10$

   $14 < 10$; -14 has the greater absolute value

   (c) $|+25| = 25$ $\quad |-8| = 8$

   $25 > 8$; $+25$ has the greater absolute value

   (d) $|+6| = 6$ $\quad |+10| = 10$

   $10 > 6$; $+10$ has the greater absolute value

   (e) $|+168| = 168$ $\quad |-150| = 150$

   $168 > 150$; $+168$ has the greater absolute value

   (f) $|-16| = 16$ $\quad |+16| = 16$

   $16 = 16$; -16 has the same absolute value as $+16$

ADDING INTEGERS

In this section you will learn about:

- adding integers

KEEPING SKILLFUL

EXERCISE 1 Be sure to show all your work.

1. $53 + 41$

2. $77 + 83$

3. $70 + 45$

4. $78 + 68$

5. $755 + 887 + 619$

6. $9385 + 5834 + 9886$

7. $958 - 474$

8. $575 - 410$

9. $9000 - 7496$

10. $5000 - 2798$

11. 2198×6

12. 1042×897

13. $52\overline{)4961}$

14. $4\overline{)500}$

15. $9\overline{)867}$

ADDITION OF INTEGERS

Because they both have magnitude and direction, integers are written with two parts; a number and a sign. For example, -7 and +2 are properly written integers.

EXERCISE 2 Represent each of the following concepts by an integer.

1. a loss of $3.00 _____
2. a gain of 4 km _____
3. a walk up 3 steps _____
4. a submarine descends 50 m _____
5. a rise of $20^{\circ}C$ _____
6. 25 km south
   (consider north the positive direction) _____
7. a climb of 1200 m _____

COMBINING INTEGERS

Ideas which can be represented by integers are often combined in everyday life.

For example, you have only $10, but you cannot resist buying a transistor radio worth $25. This puts you $15 in debt (assuming your credit is good). Fortunately, Great Aunt Matilda gives you $35 for your birthday. You pay off the debt and still have $20 in your pocket.

These transactions can be expressed as integer sentences.

original $10		spent $25		$15 in debt
↓		↓		↓
(+10)	+	(-25)	=	-15

$15 in debt		$ 35 gift		$20 left over
↓		↓		↓
(-15)	+	(+35)	=	+20

EXERCISE 3

Write a statement giving the result of the following actions. Then write an integer sentence. The first one is done for you.

1. A gain of $11 followed by a loss of $6.

   Result: a gain of five dollars

   Integer Sentence: $(+11) + (-6) = +5$

2. A gain of $6 followed by a loss of $9.

   Result: _____

   Integer Sentence: _____

3. A loss of $4 followed by a gain of $7.

   Result: _____

   Integer Sentence: _____

4. A loss of $9 followed by a loss of $2.

   Result: _____

   Integer Sentence: _____

5. Raising a flag 3 m, then lowering it 4 m.

   Result: _____

   Integer Sentence: _____

6. Lowering a flag 2 m, then raising it 1 m.

   Result: _____

   Integer Sentence: _____

7. An elevator dropping 6 storeys, going up 5 storeys, going up 8 storeys and dropping 5 storeys.

   Result: _____

   Integer Sentence: _____

EXERCISE 3 (continued)

8. An airplane dropping 300 m, then rising 120 m.

   Result: _____

   Integer Sentence: _____

9. A rise of 7°C, then a drop of 13°C.

   Result: _____

   Integer Sentence: _____

10. A rise of 17°C followed by a drop of 6°C and a drop of 9°C.

    Result: _____

    Integer Sentence: _____

What you have been doing in this exercise is adding integers. Some people prefer to call the process combining integers, since you do not always add in the usual manner.

Example

Fred Astaire, in his heyday, did a lot of dancing up and down stairs. Suppose he danced up 7 steps then down 5 steps. Then he would be 2 steps above where he started.

$$(+7) + (-5) = +2$$

Because the actions are opposite, you subtract the numbers to get your answer. The answer is given the same sign as that of the number with the greater absolute value.

In this case,

$$|+7| = 7 \qquad |-5| = 5$$

+7 has the greater absolute value. So, the answer to the addition question is +2.

ADDITION OF INTEGERS WITH LIKE SIGNS

Rule

> To add two integers with like signs, add their absolute values and put the common sign on your answer.

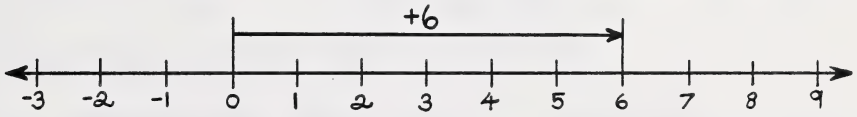
Example 1

(+6) + (+3) = ?

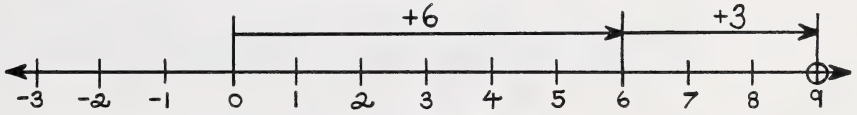
Like signs

The number line may be used to picture this operation.

1. Represent the first integer by a directed line segment (arrow) that begins at 0.



2. Represent the second integer by an arrow that begins where the first one left off.



3. Circle the integer at the end of the second arrow. This number is the sum of the two integers.

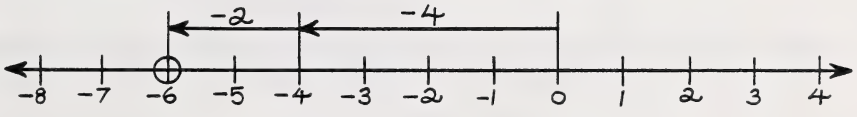
(+6) + (+3) = +9

or, following the above rule,

(+6) + (+3) = +(6 + 3)
= +9

Example 2

(-4) + (-2) = ?



(-4) + (-2) = -6

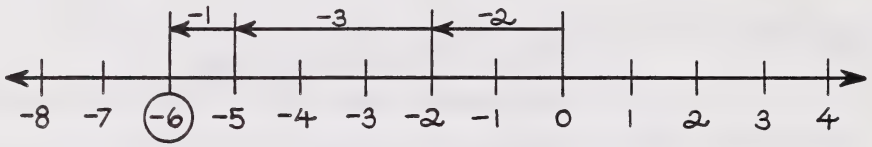
or, following the above rule,

(-4) + (-2) = -(4 + 2) = -6

Rule

To add several integers with the same sign, add their absolute values and put the common sign on your answer.

Example 3 $(-2) + (-3) + (-1) = ?$



$(-2) + (-3) + (-1)$

$= -(2 + 3 + 1)$

$= -6$

EXERCISE 4 Add each of the following sets of integers.

1. $(-6) + (-4)$

$= -(6 + 4)$

$= -10$

2. $(+6) + (+8)$

3. $(-2) + (-3)$

4. $(-84) + (-56)$

5. $(+8) + (+107)$

6. $(-111) + (-769)$

7. $(-51) + (-86) + (-4)$

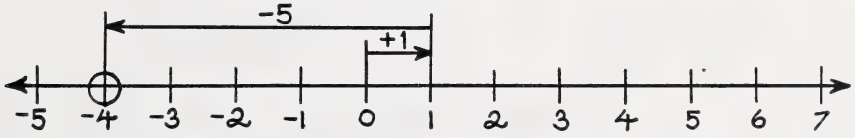
ADDITION OF INTEGERS WITH UNLIKE SIGNS

Rule

> To add integers with unlike signs, subtract their absolute values and put the sign of the integer with the greater absolute value on your answer.

Example 1 $(+1) + (-5) = ?$

This question can be pictured on a number line.



A better approach is to use the above rule.

First, determine the absolute value of each number.

$|+1| = 1$ $|-5| = 5$

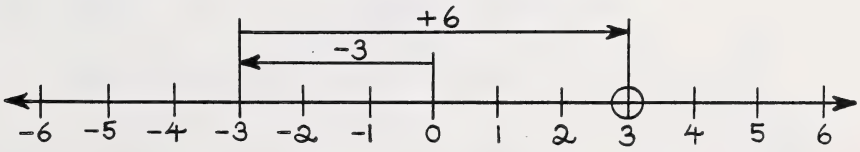
The negative number (-5) has the larger absolute value. Therefore, the answer will be negative.

Now you can subtract the absolute values to get the entire answer.

$(+1) + (-5) = -(5 - 1)$ First, determine the sign.

$= -4$ Then, subtract the absolute values.

Example 2 $(-3) + (+6) = ?$



$|-3| = 3$ $|+6| = 6$

$|+6| > |-3|$ +6 has the greater absolute value. Thus, the sign is positive.

$(-3) + (+6) = +(6 - 3) = +3$

SELF-CORRECTING EXERCISE A

Add each of the following.

1. (+8) + (-6)

2. (-3) + (+7)

3. (-2) + (+8)

4. (+4) + (-5)

5. (+19) + (-21)

6. (+6) + (-8)

Check the answers with those on page 20 of this section.
Correct any errors in your work.

EXERCISE 5 Add each of the following sets of integers.

1. (-4) + (+7)
$= +(7-4)$
$= +3$

2. (-10) + (+5)

3. (+10) + (-50)

4. (-2) + (+5)

5. (-20) + (+6)

6. (+84) + (-12)

ADDITIVE INVERSE

A special case occurs when you add two integers that are opposites. That is, they have the same magnitude (absolute value) but different direction.

Example

$(+12) + (-12)$

$|+12| = 12$ $\qquad$ $|-12| = 12$

Since the absolute values are equal you could use either sign.

$$\begin{aligned} &(+12) + (-12) \\ &= +(12 - 12) \\ &= +0 \end{aligned} \qquad \text{or} \qquad \begin{aligned} &(+12) + (-12) \\ &= -(12 - 12) \\ &= -0 \end{aligned}$$

The positive or negative sign may be used. However, zero is usually written without a sign.

Thus, $(+12) + (-12) = 0$

+12 and -12 could be said to cancel each other out.

12 and -12 are additive inverses of each other.

In general, the sum of two integers that are additive inverses is always zero.

Additive Inverse Property

> For any positive integer, a, and its additive inverse (-a):
>
> $$a + (-a) = 0$$
> $$(-a) + a = 0$$

Examples

$(+7) + (-7) = 0$

$(-11) + (+11) = 0$

$(-4) + (+4) = 0$

EXERCISE 6 Fill in the blanks below to make each statement true.

1. (a) (+4) + (-4) = ___ (b) (+6) + ___ = 0
   (c) (-5) + ___ = 0 (d) (-12) + (+12) = ___
   (e) ___ + (+15) = 0 (f) ___ + (-18) = 0

2. What is the additive inverse of each of these?
   (a) - 29 _____ (b) + 157 _____
   (c) + 37 _____ (d) - 483 _____
   (e) - 1025 _____ (f) + 89 _____
   (g) + a _____ (h) - b _____

ADDITIONS INVOLVING ZERO

Rule

> If you add zero to an integer, the integer stays the same.

Examples

(-6) + 0 = -6

0 + (-2) = -2

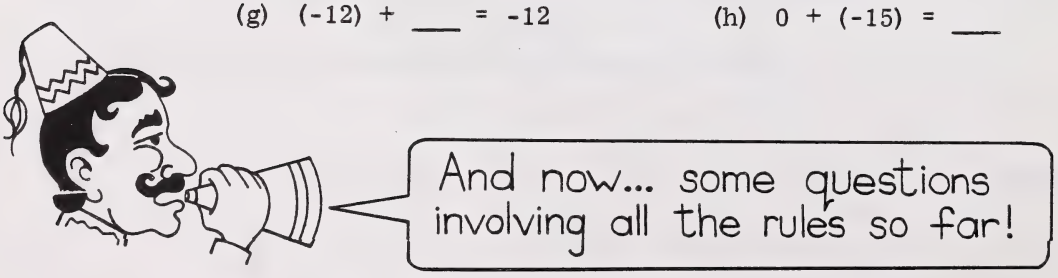
EXERCISE 7 Fill in the blanks below to make each statement true.

(a) (+4) + ___ = 4 (b) 0 + ___ = -8

(c) ___ + (-3) = -3 (d) 0 + ___ = 7

(e) ___ + (+9) = 9 (f) 0 + (+42) = ___

(g) (-12) + ___ = -12 (h) 0 + (-15) = ___



SELF-CORRECTING EXERCISE B

1. Fill in the blanks in the following chart in order to find each sum.

	Are the signs alike or different?	Must you add or subtract the absolute values?	Is the result positive or negative?	Result
(-3) + (+7)	different	subtract	positive	4
(+3) + (+6)				
(-2) + (-7)				
(+8) + (-12)				
(-9) + (-4)				
(-8) + (+5)				
(+7) + (-2)				

2. First, decide if each question represents the sum of two positive integers, the sum of two negative integers, or the sum of two integers with unlike signs. Then use the appropriate rule to find each sum.

(a) (-16) + (+5) = _____ (b) (+32) + (-8) = _____

(c) (+6) + (+4) = _____ (d) (-15) + (-2) = _____

(e) (+20) + (-30) = _____ (f) (-12) + (+12) = _____

(g) (-4) + (-18) = _____ (h) (+16) + (-4) = _____

(i) (-5) + (-7) = _____

Check the answers with those on page 20 of this section.
Correct any errors in your work.

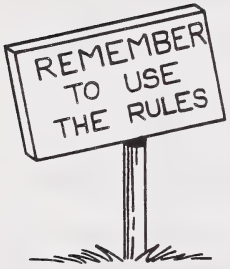
EXERCISE 8 Find each sum.

1. (+2) + (+7)

2. (-9) + (+2)

3. (+9) + (-4)

4. (-4) + (-1)



5. (-3) + (-6)

6. (+6) + (+8)

7. (+2) + 0

8. (+4) + (-4)

9. (-7) + (-8) + (-1) + (-2)

10. (+6) + (+84) + (+14) + (+1)

11. (-5) + (-89) + (-11) + (-95)

ADDITION OF MORE THAN TWO INTEGERS WITH UNLIKE SIGNS

A series of several integers may, of course, be added by adding the first two integers, then adding their sum to the third integer, and so on.

When you were adding whole numbers it was easier to add groups of ten. There is also an easier method of adding integers.

Rule

To add a series of integers:

1. add the positive integers,
2. add the negative integers,
3. use the rule for adding integers with unlike signs to find the sum of these positive and negative totals.

Example 1

$(-6) + (+5) + (-2) + (-3)$

$= (+5) + (-11)$

$= -(11 - 5)$

$= -6$

-6
-2
-3
-11

Add the three negative numbers together.

Example 2

$(+5) + (-3) + (-2) + (+6) + (+1) + (-8)$

$= (+12) + (-13)$

$= -(13 - 12)$

$= -1$

$+5$ -3
$+6$ -2
$+1$ -8
$+12$ -13

Example 3

$(-76) + (-14) + (-15) + (+6) + (+15)$

$= (-105) + (+21)$

$= -(105 - 21)$

$= -84$

-76 $+6$
-14 $+15$
-15 $+21$
-105

EXERCISE 9 Add the following.

1. (+4) + (+3) + (-8)

2. (+4) + (+3) + (-5) + (+10)

3. (-7) + (+6) + (-4) + (-5)

4. (-6) + 0 + (+7)

5. (-8) + (+7) + (-4)

6. (+2) + (-4) + (-8) + (+7)

7. (+4) + (-3) + (-2)

8. (-4) + (-2) + (-3) + (+2) + (-6)

SIMPLIFIED FORM

You may have already realized that the question,

(+6) + (+4) = +10, is the same as 6 + 4 = 10.

Similarly, (-3) + (+6) is the same as (-3) + 6 or even -3 + 6.

The addition signs are often left out and only the sign attached to the integer is left to separate the numbers.

Example 1 $-3 + 6 - 18 - 10 + 26 =$ _____

Think: This means (-3) + (+6) + (-18) + (-10) + (+26)

Proceed as usual.

$$\begin{aligned} & -3 + 6 - 18 - 10 + 26 \\ &= -31 + 32 \\ &= 1 \end{aligned}$$

$$\begin{array}{r} -3 \\ -18 \\ \underline{-10} \\ -31 \end{array} \qquad \begin{array}{r} +6 \\ \underline{+26} \\ +32 \end{array}$$

EXERCISE 10 Add the following.

1. $-3 + 5 - 8 =$

2. $18 + 6 - 4 - 10 =$

3. $-126 - 38 + 871 =$

4. -16 + 8 - 7 + 4 =

5. -4 - 8 - 9 - 13 =

6. 0 - 4 + 3 - 2 - 1 =

7. 68 - 47 - 39 + 111 =

8. -237 - 13 + 681 =

9. 75 - 79 + 60 - 66 =

10. 3841 + 412 - 83 =

SUMMARY - ADDING INTEGERS

1. Add two positive integers as you would whole numbers.

   $(+4) + (+5) = 4 + 5 = 9$

2. Add two negative integers by adding their absolute values and attaching a minus sign to the result.

   $(-4) + (-5) + -(4 + 5) = -9$

3. Add two integers with unlike signs by subtracting their absolute values and attaching the sign of the integer with the larger absolute value.

   $4 + (-5) = -(5 - 4) = -1$

   $(-4) + 5 = +(5 - 4) = +1$

4. Add more then two integers with unlike signs by adding the positive and negative integers separately, then adding the results using rule 3.

   $3 + (-9) + (-1) + 4 + (-6) + (-3) + 8$

   $= 15 + (-19)$

   $= -(19 - 15)$

   $= -4$

3	-9
4	-1
8	-6
15	-3
	-19

5. The sum of two additive inverses is zero.

   $9 + (-9) = 0$ or $9 - 9 = 0$

6. Adding 0 to an integer leaves the integer unchanged.

   $-64 + 0 = -64$

EXERCISE 11 Simplify each of the following expressions.

1. $6 + 7 =$

2. $9 + 3 =$

3. $16 + 37 =$

4. $-2 - 9 =$

5. $11 - 8 =$

6. $-37 + 42 =$

7. $3 - 4 =$

8. $-3 - 2 =$

9. $41 - 26 =$

10. $-36 - 21 =$

11. $-43 + 26 =$

12. $-13 - 14 =$

13. $-4 + 17 - 23 =$

14. $64 - 32 - 35 + 0 =$

15. $126 - 73 + 97 =$

KEY TO SELF-CORRECTING EXERCISES IN SECTION 2

SELF-CORRECTING EXERCISE A, page 9.

1. $(+8) + (-6)$
$= +(8 - 6)$
$= +2$

2. $(-3) + (+7)$
$= +(7 - 3)$
$= +4$

3. $(-2) + (+8)$
$= +(8 - 2)$
$= +6$

4. $(+4) + (-5)$
$= -(5 - 4)$
$= -1$

5. $(+19) + (-21)$
$= -(21 - 19)$
$= -2$

6. $(+6) + (-8)$
$= -(8 - 6)$
$= -2$

SELF-CORRECTING EXERCISE B, page 12.

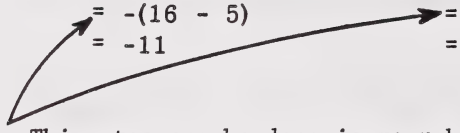
1.

	Are the signs alike or different?	Must you add or subtract the absolute values?	Is the result positive or negative?	Result
$(-3) + (+7)$	different	subtract	positive	4
$(+3) + (+6)$	alike	add	positive	9
$(-2) + (-7)$	alike	add	negative	-9
$(+8) + (-12)$	different	subtract	negative	-4
$(-9) + (-4)$	alike	add	negative	-13
$(-8) + (+5)$	different	subtract	negative	-3
$(+7) + (-2)$	different	subtract	positive	5

SELF-CORRECTING EXERCISE B, page 12.

2. (a) $(-16) + (+5)$
$= -(16 - 5)$
$= -11$

(b) $(+32) + (-8)$
$= +(32 - 8)$
$= +24$



This step can be done in your head.

(c) $(+6) + (+4)$
$= +(6 + 4)$
$= +10$

(d) $(-15) + (-2)$
$= -(15 + 2)$
$= -17$

(e) $(+20) + (-30)$
$= -(30 - 20)$
$= -10$

(f) $(-12) + (+12)$
$= 0$

(g) $(-4) + (-18)$
$= -(4 + 18)$
$= -22$

(h) $(+16) + (-4)$
$= +(16 - 4)$
$= +12$

(i) $(-5) + (-7)$
$= -(5 + 7)$
$= -12$

Key to the Exercises in Section 2

Exercise 1

1. 94
2. 160
3. 115
4. 146
5. 2261
6. 25 105
7. 484
8. 165
9. 1504
10. 2202
11. 13 188
12. 934 674
13. 95
14. 125
15. 96 R3

Exercise 2

1. -3
2. $+4$
3. $+3$
4. -50
5. $+20$
6. -25
7. $+1200$

Exercise 3

1. A gain of five dollars
   $+11+-6=+5$
2. A loss of \$3
   $+6+-9=-3$
3. A gain of \$3
   $-4++7=+3$
4. A loss of \$11
   $-9+-2=-11$
5. A loss of 1 m
   $+3+-4=-1$
6. A loss of 1 m
   $-2++1=-1$
7. A gain of 2 storeys
   $-6++5++8+-5=+2$
8. A drop of 180 m
   $-300++120=-180$
9. A drop of 6°C
   $+7+-13=-6$
10. A rise of 2°C
    $+17+-6+-9=+2$

Exercise 4

1. $(-6)+(-4)$
$=-(6+4)$
$=-10$

2. $(+6)+(+8)$
$=+(6+8)$
$=+14$

3. $(-2)+(-3)$
$=-(2+3)$
$=-5$

4. $(-84)+(-56)$
$=-(84+56)$
$=-140$

5. $(+8)+(+107)$
$=+(8+107)$
$=+115$

6. $(-111)+(-769)$
$=-(111+769)$
$=-880$

7. $(-51)+(-86)+(-4)$
$=-(51+86+4)$
$=-141$

Exercise 5

1. $(-4)+(+7)$
$=+(7-4)$
$=+3$

2. $(-10)+(+5)$
$=-(10-5)$
$=-5$

3. $(+10)+(-50)$
$=+(84-12)$
$=-40$

4. $(-2)+(+5)$
$=+(5-2)$
$=+3$

5. $(-20)+(+6)$
$=-(20-6)$
$=-14$

6. $(+84)+(-12)$
$=+(84-12)$
$=+72$

Exercise 6

1. (a) $+4+-4=0$

(b) $+6+-6=0$

(c) $-5++5=0$

(d) $-12++12=0$

(e) $-15++15=0$

(f) $+18+-18=0$

2. (a) $+29$ (b) -157

(c) -37 (d) $+483$

(e) $+1025$ (f) -89

(g) $-a$ (h) $+b$

Exercise 7

(a) $+4+0=4$ (b) $0+-8=-8$

(c) $0+-3=-3$ (d) $0++7=7$

(e) $0++9=9$ (f) $0++42=42$

(g) $-12+0=-12$ (h) $0+-15=-15$

Exercise 8

1. $+2++7=+(2+7)=+9$
2. $-9++2=-(9-2)=-7$
3. $+9+-4=+(9-4)=+5$
4. $-4+-1=-(4+1)=-5$
5. $-3+-6=-(3+6)=-9$
6. $+6++8=+(6+8)=+14$
7. $+2+0=+(2+0)=+2$
8. $+4+-4=0$
9. $-7+-8+-1+-2=-(7+8+1+2)=-18$
10. $+6++84++14++1=+(6+84+14+1)=+105$
11. $-5+-89+-11+-95=-(5+89+11+95)=-200$

Exercise 9

1. $+4++3+-8=+7+-8=7$
2. $+4++3+-5++10=+17+-5=+12$
3. $-7++6+-4+-5=+6+-16=-10$
4. $-6+0++7=-6++7=+1$
5. $-8++7+-4=-12++7=-5$
6. $+2+-4+-8++7=+9+-12=-3$
7. $+4+-3+-2=+4+-5=-1$
8. $-4+-2+-3++2+-6=-15++2=-13$

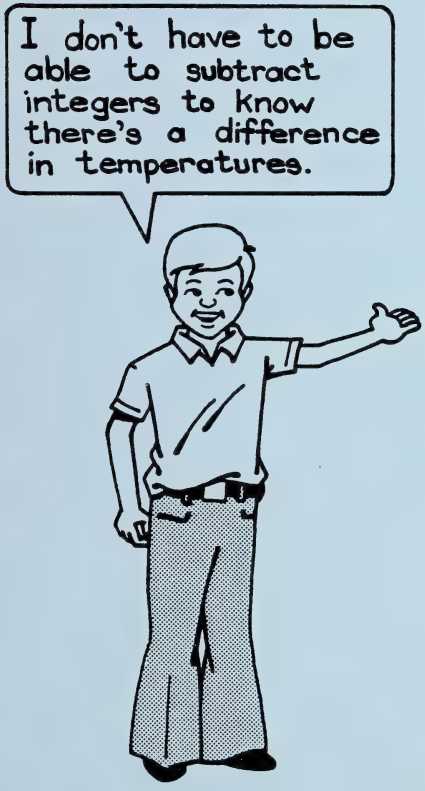
Exercise 10

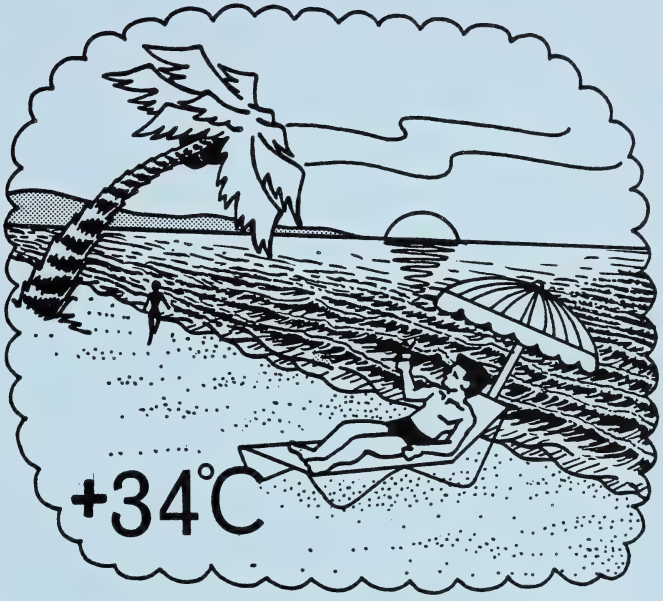
1. $-3+5+-8=-11+5=-6$
2. $18+6+-4+-10=24+-14=+10$
3. $-126+-38+871=-164+871=+707$
4. $-16+8+-7+4=-23+12=-11$
5. $-4-8-9-13=-4+-8+-9+-13=-34$
6. $0+-4+3+-2+-1=-7+3=-4$
7. $68+-47+-39+111=179+-86=+93$
8. $-237+-13+681=-250+681=+431$
9. $75+-79+60+-66=135+-145=-10$
10. $3841+412+-83=4253+-83=+4170$

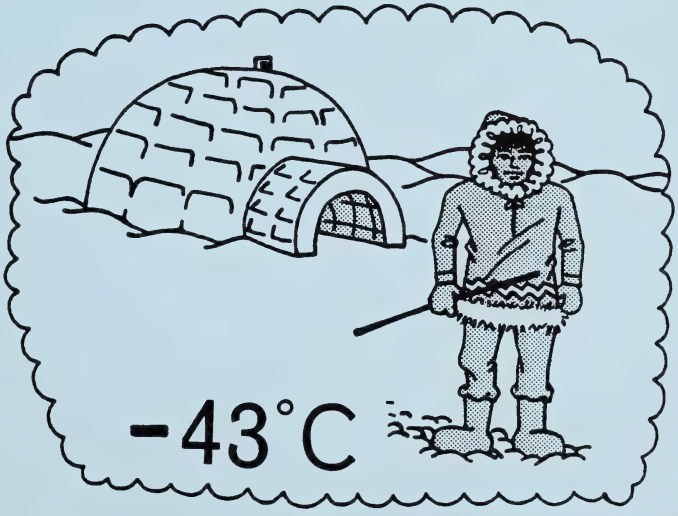
Exercise 11

1. $6+7=13$
2. $9+3=12$
3. $16+37=53$
4. $-2-9=-2+-9=-11$
5. $11-8=11+-8=3$
6. $-37+42=+5$
7. $3-4=3+-4=-1$
8. $-3-2=-3+-2=-5$
9. $41-26=41+-26=15$
10. $-36-21=-36+-21=-57$
11. $-43+26=-17$
12. $-13-14=-13+-14=-27$
13. $-4+17+-23=+17+-27=-10$
14. $64+-32+-35+0=+64+-67=-3$
15. $126+-73+97=223+-73=150$

SUBTRACTING INTEGERS







In this section you will learn about:

- finding additive inverses
- subtracting integers
- solving word problems

KEEPING SKILLFUL

EXERCISE 1

1. Arrange the following numbers in order of size, beginning with the smallest number.

   3, -6, 5, 0, -2, -7

   _____

2. Write $<$, $=$, or $>$ in each blank to make the statement true.

   (a) -6 _____ +4 (b) +3 _____ 0

   (c) -5 _____ -3 (d) +8 _____ +2

   (e) -1 _____ -8 (f) +2 _____ -7

3. Write the word greater or less in each blank to make the sentence true.

   (a) Negative numbers are always _____ than zero.

   (b) Positive numbers are always _____ than negative numbers.

   (c) Zero is always _____ than positive numbers.

4. Simplify each of the following expressions.

   (a) -17 + 15 - 17 =

   (b) 0 - 12 - 12 =

SUBTRACTING INTEGERS

Intuitive Approach

The person in the picture on the first page is correct. Not only can you easily feel the difference in temperature, you can find the difference in degrees. Simply take a thermometer like the one at the right and count the number of degrees between the two temperatures.

How much warmer is 34°C than -43°C? Counting on the thermometer you can see that there are 77° between 34°C and -43°C. Thus, 34°C is 77° warmer than -43°C.

$(+34) - (-43) = +77$ + instead of warmer

How much colder is -43°C than 34°C? The number of degrees between the two temperatures is still 77°. Thus, -43°C is 77° colder than 34°C.

$(-43) - (+34) = -77$ - instead of colder

Example

$(+6) - (+4) = 2$ but $(+4) - (+6) = ?$

Using the common interpretation of subtraction as "take away", you might conclude:

4 take away 6 leaves you 2 in the hole.

$(+4) - (+6) = -2$

General Rule

> Larger Number - Smaller Number = Positive Answer
>
> For example: $(+34) - (-43) = +77$
>
> Smaller Number - Larger Number = Negative Answer
>
> For example: $(-43) - (+34) = -77$

The above examples illustrate an intuitive method of dealing with subtraction of integers. The formal method uses a concept called the additive inverse.

ADDITIVE INVERSE

EXERCISE 2 Complete the following operations.

1. $3 - 3 =$ _____
2. $-12 + 12 =$ _____
3. $+1 - 1 =$ _____
4. $-6 + 6 =$ _____
5. $-286 + 286 =$ _____
6. $396 - 396 =$ _____

Definition

> Two numbers are additive inverses if their sum is zero.

The additive inverse of +6 is -6.

The additive inverse of -10 is +10.

The additive inverse of 0 is 0.

EXERCISE 3 Find the additive inverse of each of the following numbers.

1. 6 _____
2. -10 _____
3. -14 _____
4. 67 _____
5. 87 _____
6. 0 _____
7. -947 _____
8. 16 471 _____
9. 13 _____
10. -1 _____

11. Do all integers have additive inverses? Explain.

_____

_____

_____

FORMAL METHOD OF SUBTRACTION

Rule

To subtract a number, add its additive inverse.

Example 1

$$(+6) - (-7) = (+6) + (+7)$$

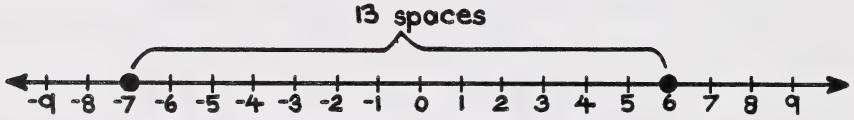
(additive inverse; change to addition)

$$= 6 + 7$$

$$= 13$$

Is this answer reasonable? Use the intuitive method to check your answer.

There are 13 spaces between +6 and -7 on the number line (or thermometer).



Since +6 is the larger number, according to the general rule on page 3, the answer should be positive. Thus, 13 seems to be a reasonable answer.

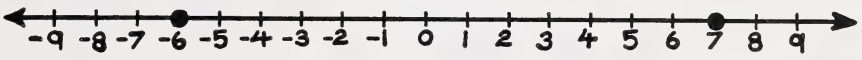
Example 2

$(-6) - (+7) = (-6) + (-7)$ (additive inverse; change to addition)

$= -6 - 7$

$= -13$

Is this answer reasonable? There are 13 spaces between -6 and +7 on the number line.



Since -6 is the smaller number, according to the general rule, the answer should be negative.

$(-6) - (+7) = -13$

As you can see, the formal method results in the same answer as the intuitive method. Since the formal method is more straight forward it will be used from now on.

You should use the intuitive method to check your answers.

Example 3

$(-10) - (-6)$ — Change -6 to +6.

$= (-10) + (+6)$ — Change subtraction to addition.

$= -10 + 6$

$= -4$

Example 4

$(+18) - (+7)$

$= (+18) + (-7)$

$= 18 - 7$

$= 11$

SELF-CORRECTING EXERCISE A

Find each of the following differences.

1. (-3) - (+1) =

2. (-2) - (+4) =

3. (-4) - (-3) =

4. (+7) - (-1) =

5. 0 - (+7) =

6. (+8) - (+4) =

7. (-2) - (-2) =

8. (+8) - (-8) =

Check your answers with those on page 20.
Correct any errors in your work.

EXERCISE 4 Perform the following subtractions.

1. (+20) - (+8) =

2. (+4) - (+19) =

3. (+10) - (+5) =

4. (+3) - (-7) =

5. (+35) - (-1) =

6. (+16) - (+16) =

7. (-7) - (+2) =

EXERCISE 4 (continued)

8. (+16) - (+6) =

9. (-3) - (+11) =

10. (-2) - (-3) =

11. (-15) - (-10) =

12. (-8) - (-2) =

13. (-8) - (-6) =

14. (-5) - (-5) =

QUESTIONS INVOLVING ADDITION AND SUBTRACTION

When you encounter questions involving both addition and subtraction, use the following procedure:

1. Change all subtractions to additions using the additive inverse.
2. Add all the positive integers.
3. Add all the negative integers.
4. Combine the totals from 2 and 3.

Example 1

Step 1 → (+6) + (+4) + (-2) + (+3) - (-8) + (-4) + (+9)

= (+6) + (+4) + (-2) + (+3) + (+8) + (-4) + (+9)

Step 2&3 → = (6 + 4 + 3 + 8 + 9) + (-2 - 4)

= 30 - 6

Step 4 → = 24

6
4
3
8
9
30

-2
-4
-6

Example 2

Step 1 → -6 - 3 + 4 - (+4) - (-2) + 6

= -6 - 3 + 4 + (-4) + (+2) + 6

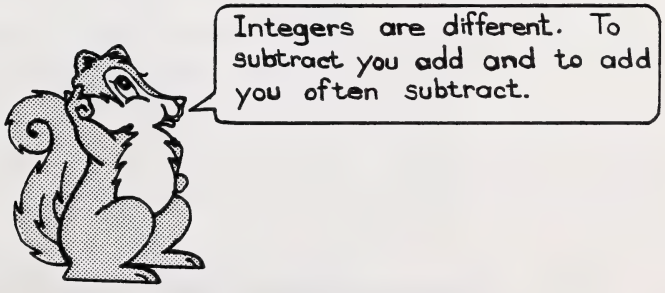
Step 2&3 → = (+4 + 2 + 6) + (-6 - 3 - 4)

= +12 - 13

Step 4 → = -1

-6
-3
-4
-13

4
2
6
12



SELF-CORRECTING EXERCISE B

Perform the following operations.

1. $(-1) + (-2) - (-3) + (-4) - (+5)$

2. $-6 + 4 - (-8) - 4 - (+7) + 6$

3. $-(+2) - 3 + 6 - (-7) - 2$

4. $8 + 4 - (-3) + 0 - 7 - (+4)$

5. $(+3) - (+2) + (-6) - (+7) + (-1)$

Check the answers with those on page 20 of this section.
Correct any errors in your work.

EXERCISE 5 Perform the following operations.

1. $4 + 3 - 8 - (-2) + 7 - 3$

2. $(-2) - (-4) + 6 - 7 - 9 + 8 + 3$

3. $12 - 3 + 4 - (-6) - 7 - 8$

4. $(-2) - 3 - (-5) + 6 - 9 + 1$

5. $3 - 4 - 6 + 7 - (-2) - (-1) - 8 + 6$

EXERCISE 6 These questions involve all the methods you have learned, for both addition and subtraction.

1. $3 - 7 =$

2. $8 - (-1) =$

3. $(-9) - (-3) =$

4. $(-7) - (+6) =$

5. $-100 - 43 =$

6. $-7 + 6 =$

7. $42 - (-16) =$

8. $-36 - 41 =$

9. $673 - (-314) =$

10. $673 - 314 =$

11. $-81 + 42 + 76 - 84 - 210 =$

12. $268 - (+345) + 871 =$

13. $6 - 4 - (-2) - 8 - (+10) + 21 + 7 =$

14. $9 + 8 - (-6) + 1 - (+4) =$

WORD PROBLEMS

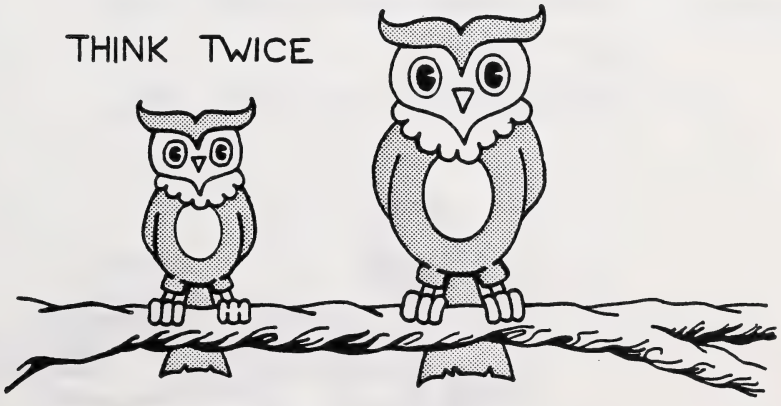
Read each problem over carefully until you understand what is being asked. It may help you to underline important words and numbers in the problem.

When you write your solution to a problem, be sure to include each of the following steps.

STEPS IN WRITING A SOLUTION

1. Tell what the variable (letter) is going to represent.
2. Write an equation using the variable and the facts of the problem.
3. Solve the equation, showing all steps and computation.
4. Write a statement answering the question asked in the problem.

After you have completed a problem, check to make sure that your answer is reasonable. Do not be like Dennis Dense who gave this answer to a problem, "The man was 237 metres high and the building was 1.8 metres tall."



APPLICATIONS

EXERCISE 7 Solve each of the following problems. Use each of the four steps listed on page 14.

1. When Mary O'Brien moved from her large house to a small apartment, she held an auction to sell some of her belongings. Based on the original cost of the items she made the following profits and losses:

   a profit of $210 on her Tiffany lamp
   a loss of $50 on her easy chair
   a loss of $10 on her bedroom rug
   a profit of $30 on her kitchen table
   a loss of $120 on her kitchen stove

   What was the overall profit or loss?

Variable — Let x represent the overall profit or loss.

Equation — $x = (+210) + (-50) + (-10) + (+30) + (-120)$

$x = +(210 + 30) + (-50 - 10 - 120)$

Solution — $x = +240 - 180$

$x = +60$

Computation

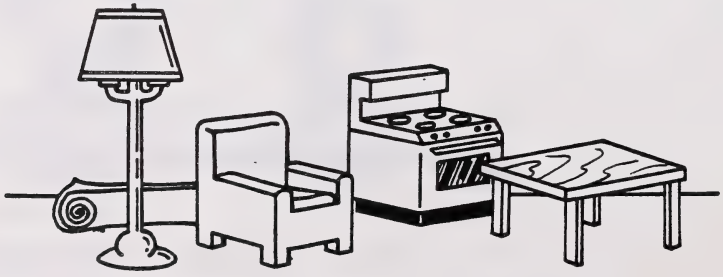
$210 + 30 = +240$

$-50 - 10 - 120 = -180$

$240 - 180 = +60$

Statement — She made an overall profit of $60.

Is your answer reasonable? yes



EXERCISE 7 (continued)

2. Marisa had $268 in the bank. She deposited her monthly cheque of $965. She then wrote a cheque for $550 for a television set and another for $45 for an antenna. She spent $85 on groceries, $400 on mortgage payments, $90 on clothing, and $45 for automobile repairs. What is left in her account?

Computation

3. In the game Peril, one may score points and receive penalties. At one stage in a game, Roger had 520 points. He then lost 250 points, gained 70 points, lost 50 points, made 520 points, lost 400 points and lost 150 points. How many points does he have now?

Computation

4. If you were to drive 30 km north and then drive 40 km south, how far would you be from your starting point? Be sure to indicate whether you are north or south of your starting point.

Computation

5. If Melbourne, Australia has a temperature of +30°C at the same time that Aklavik has a temperature of -31°C, what is the difference in their temperatures?

Computation

EXERCISE 7 (continued)

6. What is the difference in altitude between the deepest part of the Pacific Ocean (11 033 m below sea level) and the top of Mount Everest (8 848 m above sea level)?

Computation

7. The difference between the freezing point and boiling point of mercury is 395°C. If the boiling point is +357°C, what is the freezing point?

Computation

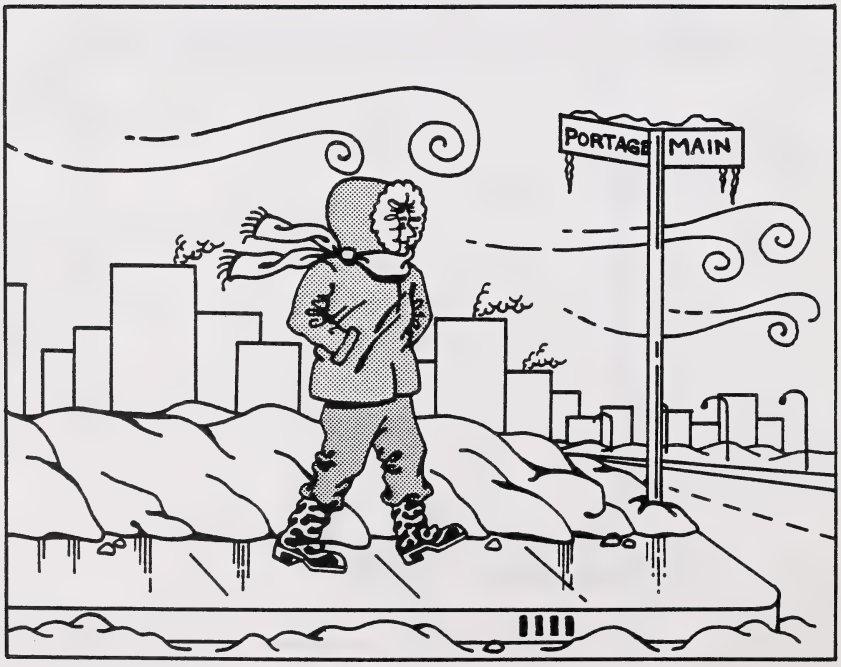
Why is a mercury thermometer not very practical outdoors in Alberta?

_____

_____

8. The record low temperature for Winnipeg was -48°C in 1969. On a very hot day in 1936 the temperature was 90°C higher. What was the temperature on that hot day?

Computation



KEY TO SELF-CORRECTING EXERCISES IN SECTION 3

SELF-CORRECTING EXERCISE A, page 7.

1. $(-3) - (+1) = (-3) + (-1) = -3 - 1 = -4$
2. $(-2) - (+4) = (-2) + (-4) = -2 - 4 = -6$
3. $(-4) - (-3) = (-4) + (+3) = -4 + 3 = -1$
4. $(+7) - (-1) = (+7) + (+1) = 7 + 1 = 8$
5. $0 - (+7) = 0 + (-7) = 0 - 7 = -7$
6. $(+8) - (+4) = (+8) + (-4) = 8 - 4 = 4$
7. $(-2) - (-2) = (-2) + (+2) = -2 + 2 = 0$
8. $(+8) - (-8) = (+8) + (+8) = 8 + 8 = 16$

SELF-CORRECTING EXERCISE B, page 11.

1. $(-1) + (-2) - (-3) + (-4) - (+5)$
   $= (-1) + (-2) + (+3) + (-4) + (-5)$
   $= +3 + (-1 - 2 - 4 - 5)$
   $= +3 - 12$
   $= -9$

2. $- 6 + 4 - (-8) - 4 - (+7) + 6$
   $= -6 + 4 + (+8) - 4 + (-7) + 6$
   $= (4 + 8 + 6) + (-6 - 4 - 7)$
   $= +18 - 17$
   $= +1$

3. $-(+2) - 3 + 6 - (-7) - 2$

$= +(-2) - 3 + 6 + (+7) - 2$

$= (6 + 7) + (-2 - 3 - 2)$

$= +13 - 7$

$= +6$

4. $8 + 4 - (-3) + 0 - 7 - (+4)$

$= 8 + 4 + (+3) + 0 - 7 + (-4)$

$= (8 + 4 + 3) + (-7 - 4)$

$= 15 - 11$

$= 4$

5. $(+3) - (+2) + (-6) - (+7) + (-1)$

$= (+3) + (-2) + (-6) + (-7) + (-1)$

$= (+3) + (-2 - 6 - 7 - 1)$

$= 3 - 16$

$= -13$

Key to the Exercises in Section 3

Exercise 1

1. $-7,\ -6,\ -2,\ 0,\ 3,\ 5$

2. (a) $-6 < +4$
   (b) $+3 > 0$
   (c) $-5 < -3$
   (d) $+8 > +2$
   (e) $-1 > -8$
   (f) $+2 > -7$

3. (a) less
   (b) greater
   (c) less

4. (a) $-17 + 15 - 17 = -34 + 15 = -19$
   (b) $0 - 12 - 12 = 0 + -12 + -12 = -24$

Exercise 2

1. $3 - 3 = 0$
2. $-12 + 12 = 0$
3. $+1 - 1 = 0$
4. $-6 + 6 = 0$
5. $-286 + 286 = 0$
6. $396 - 396 = 0$

Exercise 3

1. -6
2. $+14$
3. -87
4. $+947$
5. -13
6. $+10$
7. -67
8. 0
9. $-16\,471$
10. 1
11. Yes, both positive and negative integers proceed to infinity.

Exercise 4

1. $+20--8=+20+-8=+12$
2. $+4--19=+4+-19=-15$
3. $+10--5=+10+-5=+5$
4. $+3--7=+3++7=+10$
5. $+35--1=+35++1=+36$
6. $+16--16=+16+-16=0$
7. $-7--2=-7+-2=-9$
8. $+16--6=+16+-6=+10$
9. $-3--11=-3+-11=-14$
10. $-2--3=-2++3=+1$
11. $-15--10=-15++10=-5$
12. $-8--2=-8++2=-6$
13. $-8--6=-8++6=-2$
14. $-5--5=-5++5=0$

Exercise 5

1. $4+3-8--2+7-3=4+3+-8++2+7+-3=5$
2. $-2--4+6-7-9+8+3=-2++4+6+-7+-9+8+3=3$
3. $12-3+4--6-7-8=12+-3+4++6+-7+-8=4$
4. $-2-3--5+6-9+1=-2+-3+5+6+-9+1=-2$
5. $3-4-6+7--2--1-8+6=3+-4+-6+7+2+1+-8+6=1$

Exercise 6

1. $3-7=+3+-7=-4$
2. $8--1=8+1=9$
3. $-9--3=-9+3=-6$
4. $-7-+6=-7+-6=-13$
5. $-100-43=-100+-43=-143$
6. $-7+6=-1$
7. $42--16=42+16=58$
8. $-36-41=-36+-41=-77$
9. $673--314=673+314=987$
10. $673-314=673+-314=359$
11. $-81+42+76-84-210=-81+42+76+-84+-210=-375+118=-257$
12. $268-+345+871=268+-345+871=1139+-345=794$
13. $6-4--2-8-+10+21+7=6+-4++2+-8+-10+21+7=14$
14. $9+8--6+1-+4=9+8++6+1+-4=24+-4=20$

Exercise 7

2. $x = 268 + 965 + -550 + -45 + -85 + -400 + -90 + -45$
$= 1233 + -1215$
$= 18$

3. $x = 520 + -250 + 70 + -50 + 520 + -400 + -150$
$= 1110 + -850$
$= 260$

4. $x = 30 + -40$
$= -10$

5. $x = 30 - 31$
$= 30 + +31$
$= 61$

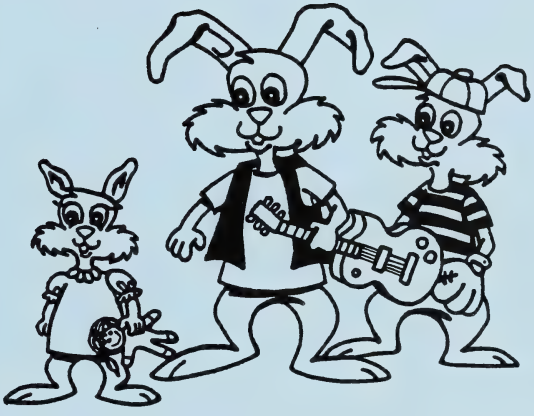
6. $x = 8848 - -11\,033$
$= 8848 + +11\,033$
$= 19\,881$

7. $x = 357 - 395$
$= 357 + -395$
$= -38$

Sometimes temperatures are below $-38\,°C$ and thus Mercury would freeze.

8. $x = -48 + 90$
$= 42$

MULTIPLYING INTEGERS



In this section you will learn about:

- multiplying integers
- solving word problems

KEEPING SKILLFUL

EXERCISE 1

1. $42 - (-16) =$
2. $-36 - 41 =$
3. $48 - 3 + 67 =$
4. $-37 - 48 =$
5. $-67 - 316 =$
6. $-4 + 67 - 37 - 106 =$
7. $-617 - (-314) =$
8. $+48 - 67 + 16 =$
9. $673 - (+214) =$
10. $43 - 73 - 66 =$
11. $346 - 176 - 189 =$
12. $-3 - (+16) - 317 =$

MULTIPLICATION OF INTEGERS

In the whole numbers, the product 3×4 means 4 added 3 times, or 3 added 4 times.

$$3 \times 4 = 4 + 4 + 4 = 12$$

$$4 \times 3 = 3 + 3 + 3 + 3 = 12.$$

Since +3 is the same as 3, and +4 is the same as 4, $(+3) \times (+4)$ is the same as 3×4.

Type 1

$$\begin{aligned}(+3) \times (+4) &= (+4) + (+4) + (+4) \\ &= 4 + 4 + 4 \\ &= 12\end{aligned}$$

Similarly,

Type 2

$$\begin{aligned}(+3) \times (-4) &= (-4) + (-4) + (-4) \\ &= -4 \ -4 \ -4 \\ &= -12\end{aligned}$$

If multiplying by +3 means adding 3 times, then multiplying by -3 means subtracting 3 times.

Type 3

$$\begin{aligned}(-3) \times (-4) &= -(-4) - (-4) - (-4) \\ &= +(+4) + (+4) + (+4) \\ &= 4 + 4 + 4 \\ &= 12\end{aligned}$$

Type 4

$$\begin{aligned}(-3) \times (+4) &= - (+4) - (+4) - (+4) \\ &= + (-4) + (-4) + (-4) \\ &= -4 - 4 - 4 \\ &= -12\end{aligned}$$

Observations

1. $(+3) \times (+4)$ and $(-3) \times (-4)$ result in the same answer.
2. $(-3) \times (+4)$ and $(+3) \times (-4)$ result in the same answer.

Rule

1. The product of two integers with the same signs is a positive integer.
2. The product of two integers with different signs is a negative integer.

Examples

1. $(+4)\ (-5)$

   $= -\ (4 \times 5)$

   $= -20$

   Different signs result in a negative product.

2. $(-5) \times (-85)$

   $+(5 \times 85)$

   $= +425$

   Like signs result in a positive product.

3. $(-3) \times (+2)$

   $= -(3 \times 2)$

   $= -6$

   This step may be done in your head.

SELF-CORRECTING EXERCISE A

Do this exercise carefully. Follow the rules and examples.

1. $-3 \times 7 =$ _____
2. $(+5)(+8) =$ _____
3. $-12 \times 6 =$ _____
4. $(-3)(8) =$ _____
5. $5 \cdot -6 =$ _____
6. $(-12)(-13) =$ _____
7. $4 \cdot -13 =$ _____
8. $(-25)(-4) =$ _____
9. $-6 \times -31 =$ _____
10. $(25)(-2) =$ _____

Check the answers with those on page 20 of this section.
Correct any errors in your work.

EXERCISE 2 Find the products.

1. $-2 \times -4 =$ _____
2. $-6 \times +1 =$ _____
3. $(-6)(-1) =$ _____
4. $-1 \times -1 =$ _____
5. $-2 \times 7 =$ _____
6. $(-6)(-7) =$ _____
7. $5 \times 4 =$ _____
8. $-12 \times -11 =$ _____
9. $(-5)(-2) =$ _____
10. $-7 \cdot -8 =$ _____
11. $6 \times -11 =$ _____
12. $(4)(-3) =$ _____
13. $+5 \cdot 6 =$ _____
14. $3 \times -8 =$ _____

For the following questions, simply do the multiplication, as usual. Then, put the proper sign on the answer. One is done for you.

15.
$$\begin{array}{r} +65 \\ -45 \\ \hline 325 \\ 2600 \\ \hline -2925 \end{array}$$

16.
$$\begin{array}{r} -60 \\ -21 \\ \hline \end{array}$$

17.
$$\begin{array}{r} -42 \\ +45 \\ \hline \end{array}$$

18.
$$\begin{array}{r} +38 \\ +40 \\ \hline \end{array}$$

19.
$$\begin{array}{r} -75 \\ -60 \\ \hline \end{array}$$

20.
$$\begin{array}{r} -66 \\ -67 \\ \hline \end{array}$$

MULTIPLICATION EQUATIONS

Example Find the integer that will replace n in $3 \times n = -21$.

Think: 3 times what number is -21?

$3 \times 7 = 21$

From the rules for multiplying integers, you know that a negative product results when two integers with different signs are multiplied.

Since 3 is positive, n must be -7.

Check: $3 \times -7 = -21$

n = -7 is correct.

SELF-CORRECTING EXERCISE B

Find the integer that can replace the letter in each of the following equations.

1. $-5 \times p = 35$

Think: $5 \times 7 = 35$
$-5 \times -7 = 35$
$p = -7$

2. $-3 \times r = 21$

3. $6 \times s = -42$

4. $-7 \times f = -21$

5. $6 \times j = 60$

6. $-4 \times k = 28$

7. $12 \times m = -48$

8. $-11 \times 2 = n$

Check the answers with those on page 20 of this section.
Correct any errors in your work.

EXERCISE 3 Find the proper value for the letter in each equation.

1. $d \times -9 = -45$
2. $e \times 5 = 55$
3. $-31 \times 4 = s$
4. $14 \times 2 = t$
5. $-11 \times h = 99$
6. $-3 \times n = 15$
7. $4 \times x = -100$
8. $-6 \times f = -60$
9. $-14 \times u = -378$
10. $-5 \times m = -150$
11. $x \times 4 = -16$
12. $y \times -3 = 81$
13. $-1 \times w = 15$
14. $k \times -1 = -8$
15. $-8 \times -7 = r$
16. $a \times 1 = 7$

MULTIPLICATION OF SEVERAL INTEGERS

To multiply several integers, simply multiply them two at a time.

Examples

1. $+4 \times -6 \times -3$

$= +4 \times +18$ ← $-6 \times -3 = +18$

$= +72$ or just 72

2. $3 \times -6 \times -4 \times -8$

$= -18 \times +32$

$= -576$

Multiply in pairs

$$\begin{array}{r} 32 \\ \times\ 18 \\ \hline 256 \\ 32 \\ \hline 576 \end{array}$$

SELF-CORRECTING EXERCISE C

1. $-2 \times 12 \times 2 \times -3 =$

2. $-12 \times 10 \times -1 =$

3. $-1 \times -2 \times -3 =$

4. $(-1)(-1)(+1)(-1)(+1) =$

Check the answers with those on page 20 of this section.
Correct any errors in your work.

EXERCISE 4 Multiply.

1. $2 \times -3 \times 6 =$

2. $5 \times 6 \times -2 =$

3. $7 \times -6 \times -3 =$

4. $-4 \times -2 \times -7 \times -1 =$

5. $6 \times -3 \times 8 =$

6. $-12 \times -6 \times -3 \times +4 =$

7. $-17 \times 26 \times -22 =$

SHORT-CUT FOR MULTIPLYING SEVERAL INTEGERS

It is often easier to decide on the sign of your answer first. Then you can go ahead and multiply the numbers without worry about the signs.

Example 1

$-2 \times 5 \times -3 \times 1 \times -4 \times -6$ ← Group negative factors together.

$= -2 \times -3 \times -4 \times -6 \times 5 \times 1$ Multiply in pairs.

$= +6 \times +24 \times +5$

$= 144 \times 5$

$= 720$

The factors were rearranged to group the negative factors together. This does not change the answer.

Since there is an even number of negative factors, they can be grouped in pairs. Each pair of negative factors gives you a positive product. Thus, the answer is positive.

You could have done the question in a shorter way.

Short Method

$-2 \times 5 \times -3 \times 1 \times -4 \times -6$

An even number of negative factors results in a positive product.

$= +(2 \times 5 \times 3 \times 1 \times 4 \times 6)$

$= 720$

Rule

> If the number of negative factors being multiplied is even, the product is positive.

Example 2

$-5 \times 3 \times -2 \times 6 \times -1$ ← Group the negative factors together.

$= -5 \times -2 \times -1 \times 3 \times 6$ Multiply in pairs.

Long Method

$= 10 \times -3 \times 6$

$= -30 \times 6$

$= -180$

When there is an odd number of negative factors, they cannot be completely paired off with each other. One is always left over to make the answer negative.

$-5 \times 3 \times -2 \times 6 \times -1$ ← An odd number of negative factors results in a negative product.

Short Method

$= -(5 \times 3 \times 2 \times 6 \times 1)$

$= -180$

Rule

> If the number of negative factors being multiplied is odd, the product is negative.

When you use this method, you do one thing at a time.

1. Decide on the sign.

   (a) An odd number of " - " signs results in a " - " product.

   (b) An even number of " - " signs results in a " + " product.

2. Multiply the numbers.

Example 3

$-2 \times -3 \times -1 \times 4$ ← 3 negative factors (an odd number of negative factors).

$= -(2 \times 3 \times 1 \times 4)$

$= -24$ ← Negative product.

Example 4

$(+5)(+3)(+2)$ ← These are simply whole numbers.

$= 5 \times 3 \times 2$

$= 30$

SELF-CORRECTING EXERCISE D

Use the short-cut rules to multiply these integers.

1. $-7 \times -11 \times 42 \times 10 =$

2. $-30 \times -40 \times -7 =$

3. $(-1)(+1)(-1)(+1)(-1) =$

Check the answers with those on page 21 of this section.
Correct any errors in your work.

EXERCISE 5 Find each of the following products.

1. $-7 \times 2 \times 3 \times -4 =$

2. $3 \times -1 \times 1 \times -1 =$

3. $(-5)(-3)(2)(+3) =$

4. $(-1)(+1)(+1)(-1)(-1) =$

5. $(4)(-2)(-3)(-1) =$

6. $-2 \times -3 \times 4 \times -1 =$

7. $3 \times -17 \times 0 \times 5 =$

REVIEW OF ADDITION, SUBTRACTION AND MULTIPLICATION

Addition Rules

1. To add integers with the same sign, add the absolute values and attach the common sign.

$$(+6) + (+4) = +(6 + 4) = +10$$

$$(-6) + (-4) = -(6 + 4) = -10$$

2. To add integers with different signs, subtract the absolute values and attach the sign of the integer with the larger absolute value.

$$(+6) + (-4) = +(6 - 4) = +2$$

$$(-6) + (+4) = -(6 - 4) = -2$$

3. To add several integers, add the negative integers, add the positive integers, then add the sums as in rule 2.

Subtraction Rule

To subtract an integer, add its additive inverse.

$$(-4) - (+3) = (-4) + (-3)$$

Multiplication Rules

1. Two factors with the same sign

   or

   Even number of negative factors

   → positive product

2. Two factors with different signs

   or

   An odd number of negative factors

   → negative product

EXERCISE 6 Evaluate each of the following expressions. Some are done for you.

1. $3 - 4 =$

2. $3 - (-4) =$

3. $-6 - 7 =$

4. $-6 - (+7) =$

5. $(-6)(+7) =$

6. $(+5)(-6)(+7) =$

7. $(-8)(6-7) = (-8)(-1)$
$= 8$

8. $(-3 + 4)(8 - 6) =$

9. $(-8)(-7)(+1)(-2)(+3) =$

10. $3 - 2 -(-6) + (-7) - 8$
$= 3 - 2 + (+6) + (-7) - 8$
$= 3 - 2 + 6 - 7 - 8$
$= 9 - 17$
$= -8$

$$\begin{array}{r} 3 \\ 6 \\ \hline 9 \end{array} \qquad \begin{array}{r} -2 \\ -7 \\ -8 \\ \hline -17 \end{array}$$

11. $-5 + 7 -(-4) - (+6) + 3$

APPLICATIONS

EXERCISE 7 Remember to follow each of these steps in solving problems:

- tell what the variable represents,
- write an equation,
- solve the equation showing all your calculations and,
- write a statement answering the question asked.

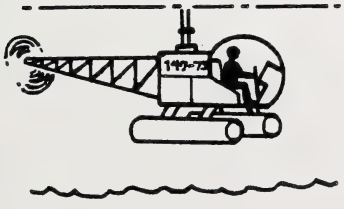
1. (a) A helicopter takes off from the surface of the Dead Sea, which is 388 metres below sea level, and climbs vertically at a rate of 4 metres per second. What is the altitude of the helicopter after one minute (60 seconds)?

Let x metres be its altitude after 60 seconds.

It will climb (4 × 60) metres in 60 seconds, but it started at -388 metres.

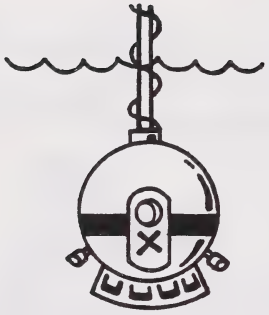
$$X = -388 + (4 \times 60)$$
$$X = -388 + 240$$
$$X = -148$$

$$\begin{array}{r} -388 \\ \underline{240} \\ -148 \end{array}$$



The helicopter will have an altitude of 148 metres below sea level after one minute.

(b) At the same time, starting at sea level, a ship lowers a bathysphere into the Mediterranean Sea at a rate of 1 metre per second. What is its altitude after one minute? Hint: it will move (-1×60) metres in 60 seconds from a starting point of 0 metres.



(c) Which will have the greater altitude, the helicopter or the bathysphere, after 60 seconds? By how many metres is it greater?

2. Allan Average has debts of $3000. From his monthly salary of $900 he needs $720 for living expenses.

(a) How much per month can he pay on his debt?

Let $x be the amount he can pay

$$x = 900 - 720$$
$$x = 180$$

He can pay $180 per month.

(b) If he pays the amount calculated in (a) each month, how much debt will remain after 12 months?

Let $y be the remaining debt

$$y = 3000 - (12 \times 180)$$
$$y = 3000 - 2160$$
$$y = 840$$

An $840 debt will remain after 12 months.

3. Metamorphosis Incorporated has present assets of $ 45 000. It is losing money at the rate of $2700 a day. What will be its financial position 30 days from now?

4. The Caspian Sea in the USSR is at an altitude of about -28 metres. La Paz, Bolivia, the highest metropolitan city in the world, is at an altitude of about 3600 metres. Which is at the greater altitude, the bottom of a 3350 metre deep mineshaft at La Paz or the top of a 280 metre radio tower on the shore of the Caspian Sea?
Hint: find the altitudes of the top of the tower and bottom of the mineshaft separately. Then compare them.
Review Question 1 on pages 16 and 17.

(a) Altitude of the top of the tower.

(b) Altitude of the bottom of the mineshaft.

(c) Comparison.

KEY TO SELF-CORRECTING EXERCISES IN SECTION 4

SELF-CORRECTING EXERCISE A, page 4.

1. $-3 \times 7 = -21$
2. $(+5)(+8) = +40$
3. $-12 \times 6 = -72$
4. $(-3)(8) = -24$
5. $5 \bullet -6 = -30$
6. $(-12)(-13) = 156$
7. $4 \times -13 = -52$
8. $(-25)(-4) = 100$
9. $-6 \bullet -31 = 186$
10. $(25)(-2) = -50$

SELF-CORRECTING EXERCISE B, page 6.

2. $-3 \times r = 21$
   $r = -7$
3. $6 \times s = -42$
   $s = -7$
4. $-7 \times f = -21$
   $f = +3$
5. $6 \times j = 60$
   $j = 10$
6. $-4 \times k = 28$
   $k = -7$
7. $12 \times m = -48$
   $m = -4$
8. $-11 \times 2 = n$
   $n = -22$

SELF-CORRECTING EXERCISE C, page 8.

1. $-2 \times 12 \times 2 \times -3$
   $= -24 \times -6$
   $= +144$
2. $-12 \times 10 \times -1$
   $= -120 \times -1$
   $= +120$
3. $-1 \times -2 \times -3$
   $= +2 \times -3$
   $= -6$
4. $(-1)(-1)(+1)(-1)(+1)$
   $= (+1)(-1)(+1)$
   $= -1$

SELF-CORRECTING EXERCISE D, page 12.

1. $-7 \times -11 \times 42 \times 10$

$= +(7 \times 11 \times 42 \times 10)$

$= +(77 \times 420)$

$= +32\ 340$

$$\begin{array}{r} 420 \\ \times\ 77 \\ \hline 2940 \\ 2940 \\ \hline 32340 \end{array}$$

2. $-30 \times -40 \times -7$

$= -(30 \times 40 \times 7)$

$= -(1\ 200 \times 7)$

$= -8400$

3. $(-1)(+1)(-1)(+1)(-1)$

$= -1$

3 negative factors result in a negative product.

Key to the Exercises in Section 4

Exercise 1

1. $42 - -16 = 42 + 16 = 58$
2. $-36 - 41 = -36 + -41 = -77$
3. $48 - 3 + 67 = 48 + -3 + 67 = 112$
4. $-37 - 48 = -37 + -48 = -85$
5. $-67 - 316 = -67 + -316 = -383$
6. $-4 + 67 + -37 + -106 = -80$
7. $-617 - -314 = -617 + +314 = -303$
8. $+48 + -67 + 16 = -3$
9. $673 + -214 = 459$
10. $43 + -73 + -66 = -96$
11. $346 + -176 + -189 = -19$
12. $-3 - +16 - 317 = -3 + -16 + -317 = -336$

Exercise 2

1. $-2 \times -4 = 8$
2. $-6 \times +1 = -6$
3. $(-6)(-1) = 6$
4. $-1 \times -1 = 1$
5. $-2 \times 7 = -14$
6. $(-6)(-7) = 42$
7. $5 \times 4 = 20$
8. $-12 \times -11 = 132$
9. $-5 \times -2 = 10$
10. $-7 \bullet -8 = 56$
11. $6 \times -11 = -66$
12. $(4)(-3) = -12$
13. $+5 \bullet 6 = 30$
14. $3 \times -8 = -24$
15. $+65 \times -45 = -2925$
16. $-60 \times -21 = +1260$
17. $-42 \times +45 = -1890$
18. $+38 \times +40 = +1520$
19. $-75 \times -60 = +4500$
20. $-66 \times -67 = +4422$

Exercise 3

1. $d \times -9 = -45$
$d = 5$

2. $e \times 5 = 55$
$e = 11$

3. $-31 \times 4 = s$
$-124 = s$

4. $14 \times 2 = t$
$28 = t$

5. $-11 \times h = 99$
$h = -9$

6. $-3 \times n = 15$
$n = -5$

7. $4 \times x = -100$
$x = -25$

8. $-6 \times f = -60$
$f = 10$

9. $-14 \times u = -378$
$u = 27$

10. $-5 \times m = -150$
$m = 30$

11. $x \times 4 = -16$
$x = -4$

12. $y \times -3 = 81$
$y = -27$

13. $-1 \times w = 15$
$w = -15$

14. $k \times -1 = -8$
$k = 8$

15. $-8 \times -7 = r$
$56 = r$

16. $a \times 1 = 7$
$a = 7$

Exercise 4

1. $2 \times -3 \times 6 = -6 \times 6 = -36$
2. $5 \times 6 \times -2 = 30 \times -2 = -60$
3. $7 \times -6 \times -3 = -42 \times -3 = 126$
4. $-4 \times -2 \times -7 \times -1 = +8 \times +7 = 56$
5. $6 \times -3 \times 8 = -18 \times 8 = -144$
6. $-12 \times -6 \times -3 \times +4 = +72 \times -12 = -864$
7. $-17 \times 26 \times -22 = -442 \times -22 = 9724$

Exercise 5

1. $-7 \times 2 \times 3 \times -4 = -14 \times -12 = 168$
2. $3 \times -1 \times 1 \times -1 = -3 \times -1 = +3$
3. $(-5)(-3)(2)(+3) = 15 \times 6 = 90$
4. $(-1)(+1)(+1)(-1)(-1) = -1$
5. $(4)(-2)(-3)(-1) = -8 \times 3 = -24$
6. $(-2 \times -3) \times (4 \times -1) = +6 \times -4 = -24$
7. $3 \times -17 \times 0 \times 5 = 0$

Exercise 6

1. $3-4=3+-4=-1$
2. $3--4=3++4=7$
3. $-6-7=-6+-7=-13$
4. $-6-+7=-6+-7=-13$
5. $(-6)(+7)=-42$
6. $(+5)(-6)(+7)=-210$
7. $(-8)(6-7)=-8\times-1=+8$
8. $(-3+4)(8-6)=1\times2=2$
9. $(-8)(-7)(+1)(-2)(+3)=-336$
10. $3+-2++6+-7+-8=-8$
11. $-5+7--4-+6+3=-5+7++4+-6+3=-11+14=+3$

Exercise 7

1. (b) $x=-1\times60$
$=-60$

(c) $x=-60--148$
$=-60+148$
$=88$

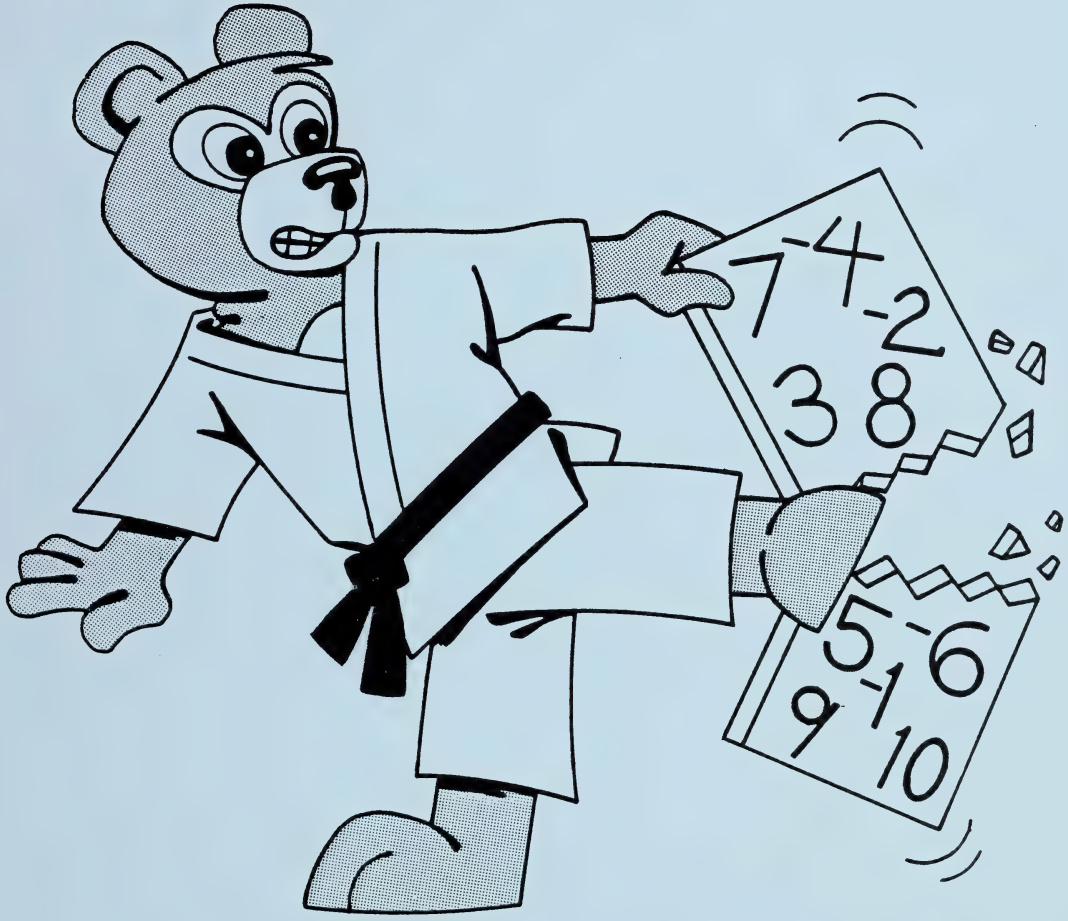
3. $x=45\,000-2700\times30$
$=45\,000-81\,000$
$=-36\,000$

4. (a) $x=280+-28$
$=252$

(b) $x=3600-3350$
$=250$

(c) The tower is 2 m higher.

DIVIDING INTEGERS



In this section you will learn about:

- dividing integers
- adding, subtracting and multiplying integers
- solving word problems

KEEPING SKILLFUL

EXERCISE 1 Complete the following operations. Show your work for Questions 1 to 4.

1. $1\frac{3}{4} + 2\frac{3}{5} + 5\frac{1}{2}$

2. $6\frac{3}{8} - 4\frac{5}{6}$

3. $\frac{3}{5} \times 2\frac{1}{4} \div 5\frac{2}{7}$

4. $82.3 \div 0.008$

5. 39.461×100

$= \_\_\_\_\_$

6. $0.0014 \div 10$

$= \_\_\_\_\_$

7. $8421 \div 1000$

$= \_\_\_\_\_$

8. $6.1 \times 10\ 000$

$= \_\_\_\_\_$

DIVISION OF INTEGERS

Division of integers is similar to division of whole numbers. However, there is an important point to keep in mind. In division of integers you must carefully consider positive and negative signs.

Example 1 $-72 \div -9 = ?$

Since division is the inverse operation of multiplication, this question is the same as:

$-9 \times ? = -72$

$9 \times 8 = 72$. So, the answer is either +8 or -8.

Try -8. $-9 \times -8 = +72$ (like signs)

Try +8. $-9 \times +8 = -72$ The answer must be +8.

Thus, $-72 \div -9 = +8$

Example 2 $-36 \div +4 = ?$

$(+4) \times ? = -36$

$(+4) \times (-9) = -36$

Thus, $-36 \div +4 = -9$.

If you were to examine more examples like this you would soon see a pattern.

Rule

1. The quotient of integers with like signs is positive.
2. The quotient of integers with unlike signs is negative.

Note that this set of sign rules is the same as that for multiplication.

Further Examples

(a) $-8 \div +2 = -4$

(b) $-16 \div -4 = +4$

(c) $+36 \div -9 = -4$

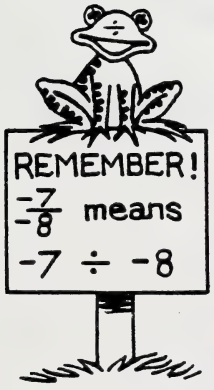
(d) $+20 \div +5 = +4$

Part (d) is the same as $20 \div 5 = 4$.

EXERCISE 2 Perform each of the following divisions.

1. $-15 \div -5 =$ _____ 2. $24 \div -12 =$ _____

3. $-12 \div 6 =$ _____ 4. $64 \div -16 =$ _____



5. $\frac{-18}{-9} =$ _____ 6. $\frac{0}{-3} =$ _____

7. $\frac{9}{-9} =$ _____ 8. $\frac{-30}{-10} =$ _____

9. $-84 \div 12 =$ _____ 10. $-876 \div 1 =$ _____

11. $110 \div -11 =$ _____ 12. $-195 \div -5 =$ _____

13. $\frac{-24}{8} =$ _____ 14. $\frac{-48}{-16} =$ _____

USING DIVISION OF INTEGERS TO SOLVE EQUATIONS

In Lesson 7 you learned to solve equations of the form $ax = b$. You solved them using inverse operations.

Example

$3x = 18$

$x = 18 \div 3$ ← *Multiplication becomes division (inverse operation).*

$x = 6$

Now that you know how to divide integers, you can solve the same kind of equations even when integers are involved.

Example 1

$-5x = -10$ ← *-5x means -5 times x.*

$x = -10 \div -5$ ← *Inverse operations.*

$x = 2$

Check:

$-5x = -10$	
$-5(2)$	-10
-10	✓

Example 2

$-x = 7$ ← *Recall that -x means -1x.*

$-1x = 7$

$x = 7 \div -1$

$x = -7$

Check:

$-x = 7$	
$-(-7)$	7
$(-1)(-7)$	
7	✓

EXERCISE 3 Solve each of the following equations. Check your solutions.

1. $-5x = 15$ Check:

2. $3w = -12$ Check:

3. $-t = -9$ Check:

4. $-9m = -108$ Check:

5. $8k = 456$ Check:

6. $-y = 9684$ Check:

7. $-5m = -1965$ Check:

OPERATIONS USING INTEGERS

You have now studied the four basic operations using integers: addition, subtraction, multiplication, and division.

Here is a summary of the sign rules.

Addition

1. To add two integers with like signs, add their absolute values and attach the common sign.

$$(-6) + (-4) = -(6 + 4) = -10$$

2. To add two integers with unlike signs, subtract their absolute values and attach the sign of the integer with the larger absolute value.

$$(-6) + (+4) = -(6 - 4) = -2$$

Subtraction

To subtract two integers, add the additive inverse of the second integer.

$$(-8) - (-4) = (-8) + (+4) = -4$$

Multiplication and Division

1. Like signs result in a positive answer.

$$-6 \times -4 = +24 \qquad -6 \div -3 = +2$$

2. Unlike signs result in a negative answer.

$$-6 \times +4 = -24 \qquad -6 \div +3 = -2$$

Multiplication of Several Factors

When several integers are multiplied together:

(a) The product is positive if the number of negative factors is even.

(b) The product is negative if the number of negative factors is odd.

$(-2)\ (+2)\ (-2)\ (+2) = +16$ ← *2 negative factors*

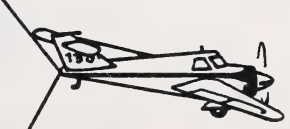
$(-2)\ (-2)\ (-2)\ (+2) = -16$ ← *3 negative factors*

Combinations of Addition and Subtraction

1. Change the subtraction to addition by using the additive inverse.
2. Add all the positive integers.
3. Add all the negative integers.
4. Combine the totals from steps 2 and 3.

$$(+4) - (+8) + (-5) - (-7)$$
$$= (+4) + (-8) + (-5) + (+7) \quad \textit{Step 1}$$
$$= (4 + 7) + (-8 - 5)$$
$$= 11 - 13 \quad \textit{Steps 2 and 3}$$
$$= -2 \quad \textit{Step 4}$$

PAGES 7 and 8 CONTAIN ALL THE RULES FOR INTEGER OPERATIONS



SELF-CORRECTING EXERCISE A

Put the word "positive" or the word "negative" in each of the following blanks.

1. The sum of two negative integers is a _____ integer.

2. The product of two negative integers is a _____ integer.

3. The difference of two negative integers might be either a _____ or a _____ integer, depending on the order of the subtraction.

4. The additive inverse of a positive integer is a _____ integer.

5. The quotient of two positive integers is a _____ integer.

6. The product of a positive integer and a negative integer is a _____ integer.

7. The quotient of a _____ integer and a negative integer is a negative integer.

8. The sum of a positive integer and a negative integer is a _____ integer if it is the negative integer that has the larger absolute value.

9. The product of an odd number of negative integers is a _____ integer.

10. The sum of zero and a positive integer is a _____ integer.

Check the answers with those on page 21 of this section.
Correct any errors in your work.

EXERCISE 4 Perform each of the following operations.

1. $(+2) + (-1) =$ _____

2. $(+24) + (-37) =$ _____

3. $-3 - 6 =$ _____

4. $-21 + 14 =$ _____

5. $(-21) - (-3) =$ _____

6. $(-65) - (+64) =$ _____

7. $-64 - 23 + 5 =$ _____

8. $6 - 4 - 6 =$ _____

9. $(-3) - (-2) + (+4) =$ _____

10. $(-4) + (-6) - (+4) + (-4) + (-8) - (-3) =$ _____

11. $(-1)(+3) =$ _____

12. $(-2)(-6) =$ _____

13. $(-6)(0) =$ _____

14. $(-6)(-2)(+2) =$ _____

15. $(-8)(-6)(-13) =$ _____

16. $(-16)(+3)(-21) =$ _____

17. $(+10) \div (-2) =$ _____

18. $(-15) \div (+3) =$ _____

19. $\frac{-36}{9} =$ _____

20. $\frac{-49}{-7} =$ _____

21. $\frac{100}{-10} =$ _____

WORKING WITH SEVERAL OPERATIONS

When a question involves several operations, the rules for order of operations tell you which operation to do first. In previous work you will have studied order of operations rules for whole numbers and fractions. These same rules apply to integers as well.

Order of Operations

> Step 1 - Do all calculations in brackets first.
>
> Step 2 - Multiply or divide in the order the operations appear (left to right).
>
> Step 3 - Add or subtract in the order the operations appear (left to right).

Example 1

$$[(-45) \div (-5)] \times [27 \div (-9)]$$
$$= [+9] \times [-3]$$
$$= -27$$

Example 2

$$(-3) \times [18 + (-5)]$$
$$= (-3) \times [13]$$
$$= -39$$

Example 3

$$(-4) + 16 \div (-4)$$

Do this operation first.

$$= -4 - 4$$
$$= -8$$

Example 4

$$\frac{(-1) + (-2)(+3)}{(-7) \times (-2) \div (-2)}$$

$$= \frac{(-1) + (-6)}{(+14) \div (-2)}$$

Simplify the numerator and denominator separately.

$$= \frac{-7}{-7}$$
$$= +1$$

SELF-CORRECTING EXERCISE B

Evaluate the following number expressions.

1. $[(-4) + 16] \div (-4) =$

2. $(-8) \times (-9) + 5 =$

3. $(-8) \times [(-9) + 5] =$

4. $(-4)(-2) + (-2) =$

5. $\dfrac{(-8) + (-4)}{(-8) - (-4)} =$

6. $\dfrac{-56 + 21}{(-5) \cdot (-7)} =$

7. $\dfrac{(-3) + (-3)(+4)}{(-6) \times (+5) \div (-2)} =$

Check the answers with those on page 21 of this section.
Correct any errors in your work.

EXPRESSIONS WITH EXPONENTS

In Lesson 4 you learned how to evaluate powers of whole numbers.

For example, you learned that:

$3^4 = (3)(3)(3)(3) = 81.$

4 factors of 3.

Similarly,

$(-3)^4 = (-3)(-3)(-3)(-3) = 81$

4 negative factors.

$(-2)^3 = (-2)(-2)(-2) = -8$

3 negative factors.

These powers can be combined in number expressions.

Example 1

$(-6)^2 - (-2)^3$

$= (-6)(-6) - (-2)(-2)(-2)$

$= (+36) - (-8)$

$= (+36) + (+8)$

$= 44$

Example 2

$3(-2)^2 - 7$

$= 3(-2)(-2) - 7$ ← *Note that the 3 is not squared.*

$= 12 - 7$

$= 5$

SELF-CORRECTING EXERCISE C

Evaluate the following expressions.

1. $(-6)^2$ =

2. $(-4)^2$ =

3. $(-3)^3$ =

4. $(-5)^3$ =

5. $(-3)^2 - (-5)^2$ =

6. $3(-2)^2 + 2(-3)^2$ =

7. $(-1)^6$ =

8. $(-3)^2 + 5$ =

Check the answers with those on page 22 of this section.
Correct any errors in your work.

EXERCISE 5 Evaluate each of the following expressions.

1. $(-6) + (-12) \div (-2) =$

2. $(-3)(-7) + 2 =$

3. $\dfrac{-10 - 32}{-3} =$

4. $\dfrac{8 - (-36) \div 6}{(-49) \div 7} =$

5. $(-8) \div (-4) + (-6) \div (-2) =$

6. $(-8) - [(-6) - 3] - [(-2) + 6] =$

7. $8^2 - 7^2 =$

8. $(-2)^2 - (-5)^2 =$

9. $3(-5)^2 \div (-15) =$

PROBLEMS INVOLVING INTEGERS

The problems in this section will be very similar to those in Exercise 5. The biggest difference is that they will be written in words.

Before you start, be sure you know what the words sum, difference, product, quotient and square mean. Use a dictionary, if necessary.

Example 1 Subtract the sum of -14 and +19 from -20.

Method 1 First, find the sum of -14 and +19.

$$\begin{aligned} &(-14) + (+19) \\ &= -14 + 19 \\ &= 5 \end{aligned}$$

Then subtract the sum from -20.

$$\begin{aligned} &(-20) - (+5) \\ &= (-20) + (-5) \\ &= -20 - 5 \\ &= -25 \end{aligned}$$

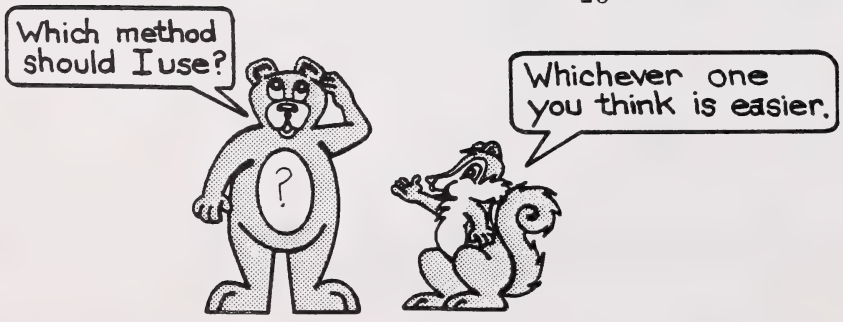
The result is -25.

Example 1 could also have been done this way.

Method 2

$$\begin{aligned} &(-20) - [(-14) + (+19)] \\ &= (-20) - (+5) \\ &= -25 \end{aligned}$$

The operation that must be done first is put in brackets.



Example 2 Divide the product of -8 and -9 by 4.

Method 1 First, find the product of -8 and -9.

$$(-8)(-9) = +72$$

Then, divide it by 4.

$$72 \div 4 = 18$$

The result is 18.

Method 2

$$[(-8)(-9)] \div 4$$
$$= +72 \div 4$$
$$= 18$$

EXERCISE 6 Solve the following, using either method shown on pages 16 and 17. Show all your calculations.

1. Add -50 to the sum of -79 and 97.

2. Divide the sum of -113, 95, and -21 by -13.

3. Subtract the sum of - 27 and 16 from - 32.

4. Find the sum of - 69 and 33, than subtract - 17 from it.

5. Find the product of - 6 and 9, then divide this amount into - 216.

6. Divide the product of - 15 and - 7 by 5.

7. Divide $(-4)^2$ by $(-2)^2$.

8. Subtract -936 from 1982, then divide this difference by -2.

9. Add the square of -2 to the square of 5.

10. Square the product of -2 and +8.

KEY TO SELF-CORRECTING EXERCISES IN SECTION 5

SELF-CORRECTING EXERCISE A, page 9.

1. negative
2. positive
3. negative positive
4. negative
5. positive
6. negative
7. positive
8. negative
9. negative
10. positive

SELF-CORRECTING EXERCISE B, page 13.

1. $[(-4) + 16] \div (-4)$
   $= [+12] \div (-4)$
   $= -3$

2. $(-8) \times (-9) + 5$
   $= +72 + 5$
   $= 77$

3. $(-8) \times [(-9) + 5]$
   $= (-8) \times [-4]$
   $= 32$

4. $(-4)(-2) + (-2)$
   $= (+8) + (-2)$
   $= 6$

5. $\dfrac{(-8) + (-4)}{(-8) - (-4)}$
   $= \dfrac{-12}{(-8) + (+4)}$
   $= \dfrac{-12}{-4}$
   $= +3$

6. $\dfrac{-56 + 21}{(-5) \cdot (-7)}$
   $= \dfrac{-35}{+35}$
   $= -1$

7. $\dfrac{(-3) + (-3)(+4)}{(-6) \times (+5) \div (-2)}$
   $= \dfrac{(-3) + (-12)}{(-30) \div (-2)}$
   $= \dfrac{-15}{+15}$
   $= -1$

SELF-CORRECTING EXERCISE C, page 15.

1. $(-6)^2$
$= (-6)(-6)$
$= 36$

2. $(-4)^2$
$= (-4)(-4)$
$= 16$

3. $(-3)^3$
$= (-3)(-3)(-3)$
$= -27$

4. $(-5)^3$
$= (-5)(-5)(-5)$
$= -125$

5. $(-3)^2 - (-5)^2$
$= (-3)(-3) - (-5)(-5)$
$= (+9) - (+25)$
$= 9 - 25$
$= -16$

6. $3(-2)^2 + 2(-3)^2$
$= 3(-2)(-2) + 2(-3)(-3)$
$= 12 + 18$
$= 30$

7. $(-1)^6$
$= (-1)(-1)(-1)(-1)(-1)(-1)$
$= +1$

8. $(-3)^2 + 5$
$= (-3)(-3) + 5$
$= 9 + 5$
$= 14$

Key to the Exercises in Section 5

Exercise 1

1. $1\frac{3}{4}+2\frac{3}{5}+5\frac{1}{2}$
$=1\frac{15}{20}+2\frac{12}{20}+5\frac{10}{20}$
$=8\frac{31}{20}$
$=9\frac{17}{20}$

2. $6\frac{3}{8}-4\frac{5}{6}$
$=6\frac{9}{24}-4\frac{20}{24}$
$=5\frac{33}{24}-4\frac{20}{24}$
$=1\frac{13}{24}$

3. $\frac{3}{5}\times 2\frac{1}{4}\div 5\frac{2}{7}$
$=\frac{3}{5}\times\frac{9}{4}\div\frac{37}{7}$
$=\frac{3}{5}\times\frac{9}{4}\times\frac{7}{37}$
$=\frac{189}{740}$

4. $82.3\div 0.008 = 10\,287.5$

5. $39.461\times 100 = 3946.1$

6. $0.0014\div 10 = 0.00014$

7. $8421\div 1000 = 8.421$

8. $6.1\times 10\,000 = 61\,000$

Exercise 2

1. $-15\div -5=+3$
2. $24\div -12=-2$
3. $-12\div 6=-2$
4. $64\div -16=-4$
5. $\frac{-18}{-9}=2$
6. $\frac{0}{-3}=0$
7. $\frac{9}{-9}=-1$
8. $\frac{-30}{-10}=+3$
9. $-84\div 12=-7$
10. $-876\div 1=-876$
11. $110\div -11=-10$
12. $-195\div -5=39$
13. $\frac{-24}{8}=-3$
14. $\frac{-48}{-16}=3$

Exercise 3

1. $-5x = 15$
   $x = -3$

2. $3w = -12$
   $w = -4$

3. $-t = -9$
   $t = 9$

4. $-9m = -108$
   $m = 12$

5. $8k = 456$
   $k = 57$

6. $-y = 9684$
   $y = -9684$

7. $-5m = -1965$
   $m = 393$

Exercise 4

1. $+2 + -1 = +1$

2. $+24 + -37 = -13$

3. $-3 - 6 = -3 + -6 = -9$

4. $-21 + 14 = -7$

5. $-21 - -3 = -21 + 3 = -18$

6. $-65 - +64 = -65 + -64 = -129$

7. $-64 + -23 + 5 = -82$

8. $6 + -4 + -6 = -4$

9. $-3 + +2 + +4 = 3$

10. $-4 + -6 + -4 + -4 + -8 + +3 = -23$

11. $(-1)(+3) = -3$

12. $(-2)(-6) = 12$

13. $(-6)(0) = 0$

14. $(-6)(-2)(+2) = 24$

15. $(-8)(-6)(-13) = -624$

16. $(-16)(+3)(-21) = 1008$

17. $+10 \div -2 = -5$

18. $-15 \div +3 = -5$

19. $\frac{-36}{9} = -4$

20. $\frac{-49}{-7} = 7$

21. $\frac{100}{-10} = -10$

Exercise 5

1. $-6+(-12)\div(-2)=-6+6=0$

2. $(-3)(-7)+2=21+2=23$

3. $\dfrac{-10-32}{-3}=\dfrac{-10+-32}{-3}=\dfrac{-42}{-3}=+14$

4. $\dfrac{8--36\div 6}{-49\div 7}=\dfrac{8--6}{-7}=\dfrac{14}{-7}=-2$

5. $-8\div-4+-6\div-2=2+3=5$

6. $-8-[-6+-3]-[-2+6]=-8++9+-4=-3$

7. $8^2-7^2=8\times 8-7\times 7=64-49=15$

8. $(-2)^2-(-5)^2=-2\times-2--5\times-5=+4-+25=-21$

9. $3(-5)^2\div-15=3\times-5\times-5\div-15=75\div-15=-5$

Exercise 6

1. $-79+98+-50=-129+97=-32$

2. $\dfrac{-113+95+-21}{-13}=\dfrac{-134+95}{-13}=\dfrac{-39}{-13}=+3$

3. $-32-(-27+16)=-32--11=-32++11=-21$

4. $(-69+33)--17=-36++17$
$=-19$

5. $-216\div(-6\times 9)=-216\div-54=4$

6. $(-15 \times -7) \div 5 = 105 \div 5 = 21$

7. $(-4)^2 \div (-2)^2 = \dfrac{-4 \times -4}{-2 \times -2} = \dfrac{+16}{+4} = +4$

8. $\dfrac{1982 - -936}{-2} = \dfrac{1982 + 936}{-2} = \dfrac{2918}{-2} = -1459$

9. $(-2)^2 + (5)^2 = 4 + 25 = 29$

10. $(-2 \times +8)^2 = (-16)^2 = -16 \times -16 = +256$

FRACTIONS

In this section you will learn about:

- reducing fractions
- changing improper fractions to mixed numbers
- calculating the LCM
- writing equivalent fractions
- adding and subtracting fractions and mixed numbers

KEEPING SKILLFUL

EXERCISE 1

1. Complete the following by putting one of these signs ($>$, $<$, $=$) between the given numbers.

   (a) 5 ____ -5 (b) 0 ____ -2

   (c) -12 ____ -3 (d) -32 ____ 7

2. Perform the following operations.

   (a) $-6 + 8 =$ (b) $-9 + 14 =$

   (c) $8 - (-14) =$ (d) $-21 - (-7) =$

3. Evaluate the following.

   (a) $3^4 =$ (b) $7^3 =$

   (c) $10^5 =$ (d) $13^2 =$

4. Use the Product of Powers property to evaluate the following. Write your answer in standard form.

   $2^5 \times 2^2 \times 2 =$

REDUCING FRACTIONS

A fraction can be reduced by dividing both the numerator and the denominator by common factors. When the numerator and denominator have no factor in common, except 1, the fraction is said to be in lowest terms.

Example 1 Reduce $\frac{30}{36}$ to lowest terms.

What number divides into 30 and 36? 2 will.

$$\frac{30}{36} \overset{\div 2}{=} \frac{15}{18}$$

What number divides into 15 and 18? 3 will.

$$\frac{15}{18} \overset{\div 3}{=} \frac{5}{6}$$

What number divides into 5 and 6? Only 1 will.

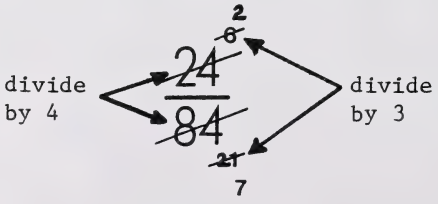
Thus $\frac{30}{36}$ in lowest terms is $\frac{5}{6}$.

Example 2 Reduce $\frac{24}{84}$ to lowest terms.

$$\frac{24}{84} \overset{\div 2}{=} \frac{12}{42} \overset{\div 2}{=} \frac{6}{21} \overset{\div 3}{=} \frac{2}{7}$$

Notice that you could have divided by 4 and 3, or by 12 which is the greatest common factor.

Most mathematicians save time and space by showing their reducing in this manner.



EXERCISE 2 Reduce the following fractions to lowest terms.

1. $\frac{10}{65} = \frac{\overset{2}{\cancel{10}}}{\underset{13}{\cancel{65}}} = \frac{2}{13}$

2. $\frac{16}{64} =$

3. $\frac{24}{36} =$

4. $\frac{8}{48} =$

5. $\frac{16}{32} =$

6. $\frac{22}{121} =$

7. $\frac{14}{56} =$

8. $\frac{48}{52} =$

9. $\frac{30}{55} =$

10. $\frac{20}{110} =$

11. $\frac{117}{130} =$

12. $\frac{99}{132} =$

CHANGING IMPROPER FRACTIONS TO MIXED NUMBERS

An **improper fraction** has a numerator which is greater than or equal to its denominator. $\frac{5}{3}$, $\frac{8}{8}$, and $\frac{12}{11}$ are examples of improper fractions.

A **mixed number** contains a natural number plus a fraction. $2\frac{1}{3}$, $30\frac{1}{2}$ and $18\frac{3}{4}$ are examples of mixed numbers.

Recall that a fraction, $\frac{a}{b}$, is another way of showing division. $\frac{a}{b}$ means the same as $a \div b$

Rule

> To change an improper fraction to a mixed number, divide the denominator into the numerator.

Example Write $\frac{20}{6}$ as a mixed number in lowest terms.

$$\frac{20}{6} = 3\frac{2}{6}$$

$$= 3\frac{1}{3}$$

$$\begin{array}{r} 3 \\ 6\overline{)\,20} \\ \underline{18} \\ 2 \end{array}$$

← The quotient is the natural number part of the mixed number.

← The remainder placed over the divisor is the fractional part of the mixed number.

Notice that the fractional part of the mixed number $\frac{2}{6}$, equals $\frac{1}{3}$ in lowest terms.

EXERCISE 3 Change the following improper fractions to mixed numbers in lowest terms.

1. (a) $\frac{22}{7} =$ _____ (b) $\frac{37}{10} =$ _____

(c) $\frac{17}{5} =$ _____ (d) $\frac{23}{8} =$ _____

(e) $\frac{26}{12} =$ _____ (f) $\frac{28}{8} =$ _____

PRIME FACTORIZATION

Prime numbers have exactly two factors, one and the number itself. The set of prime numbers is {2, 3, 5, 7, 11, 13, 17, 19, 23...}.

Method

> To find the prime factors of a number:
>
> (a) divide the original number and each of the resulting quotients by the smallest prime number possible,
>
> (b) continue this process until a quotient of 1 is obtained. The divisors are the prime factors.

This method requires you to check if any of the prime numbers divide into the given number. The set of prime numbers is given above. Here are some easy ways for checking if the first three primes divide into the given number.

Divisibility by 2, 3 and 5

A number is divisible by two if it ends in the even numbers 0, 2, 4, 6 or 8.

A number is divisible by three, if the sum of the digits is divisible by 3. For example, the number 2214 is divisible by 3, because $2 + 2 + 1 + 4 = 9$ and 9 is divisible by 3.

A number is divisible by five if it ends in either 0 or 5.

Example

Write 60 as the product of its prime factors.

$$\begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ \hline & 1 \end{array}$$

Notice that the division is done in a downhill format.
The divisors are 2, 2, 3 and 5.
They are the prime factors of 60.
60 is equal to the product of its prime factors.

$60 = 2 \times 2 \times 3 \times 5$

REDUCING FRACTIONS USING PRIME FACTORIZATION

Page 3 of this lesson explained the most common method for reducing fractions. However, finding common factors of the numerator and denominator can be quite difficult. Another approach is to write both the numerator and denominator as products of their prime factors. Then the common factors are easily seen and you simply cancel them out.

Example Write $\frac{42}{56}$ in lowest terms.

$\frac{42}{56} = \frac{2 \times 3 \times 7}{2 \times 2 \times 2 \times 7}$ Write the numerator and denominator as products of their prime factors.

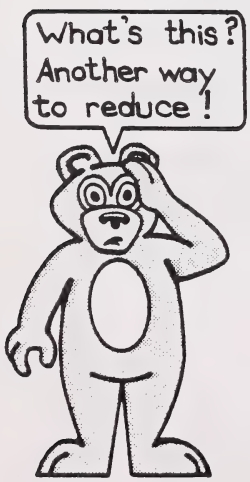
2	42
3	21
7	7
	1

$= \frac{\cancel{2} \times 3 \times \cancel{7}}{\cancel{2} \times 2 \times 2 \times \cancel{7}}$ Cancel out the common factors.

2	56
2	28
2	14
7	7
	1

$= \frac{3}{4}$ The product of the remaining factors is the fraction in lowest terms.

EXERCISE 4 Reduce the following fractions to lowest terms.



1. $\frac{14}{63} =$

2. $\frac{10}{105} =$

3. $\frac{64}{256} =$

4. $\frac{34}{51} =$

LEAST COMMON MULTIPLE (LCM)

Another use of prime factorization is finding the least common multiple of two or more numbers.

Method

> To find the least common multiple (LCM) of several numbers:
>
> (a) write each number as a product of its prime factors.
>
> (b) list each prime factor the greatest number of times it occurs in any one number, and
>
> (c) find the product of these factors.

Example 1 Find the LCM of 56 and 126.

First, do the prime factorization.

$$56 = 2 \times 2 \times 2 \times 7$$
$$126 = 2 \times 3 \times 3 \times 7$$

Then, list the greatest number of times each factor occurs. (The factors in the given numbers are 2, 3, 7.)

$56 = (2 \times 2 \times 2) \times (7)$ ← Largest group of twos. Seven appears once in each number. The largest group of sevens contains one seven.

$126 = 2 \times (3 \times 3) \times 7$ ← Largest group of threes.

Now, find the product of these factors.

$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 7$$
$$= 504$$

Example 2 Find the LCM of 3, 30 and 125.

Prime factorization:

$$3 = 3$$
$$30 = 2 \times 3 \times 5$$
$$125 = 5 \times 5 \times 5$$

In the largest group of each factor:
- the factor 3 occurs *once*
- the factor 2 occurs *once*
- the factor 5 occurs *three* times

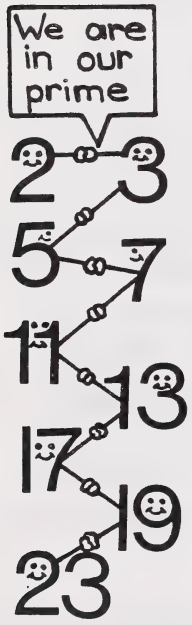
$$\text{LCM} = 2 \times 3 \times 5 \times 5 \times 5$$
$$= 750$$

SELF-CORRECTING EXERCISE A

Write each number as the product of its prime factors.
Then find the least common multiple of the numbers listed.

1. 4 and 6

2. 8 and 24



3. 5 and 9

4. 3, 6, and 10

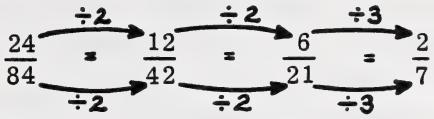
5. 42 and 90

6. 105 and 360

Check the answers with those on page 21 and 22 of this section.
Correct any errors in your work.

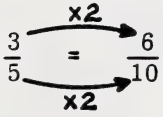
EQUIVALENT FRACTIONS

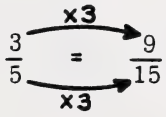
On page 3 of this lesson, the fraction $\frac{24}{84}$ was reduced in the manner shown below.

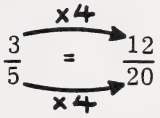


Each of the fractions $\frac{12}{42}$, $\frac{6}{21}$, and $\frac{2}{7}$ is equivalent to $\frac{24}{84}$. Notice that each of these fractions can be formed by dividing both the numerator and denominator of $\frac{24}{84}$ by the same number.

Another way of forming an equivalent fraction is by multiplying both the numerator and denominator by the same number.







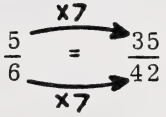
Each of the fractions $\frac{6}{10}$, $\frac{9}{15}$, and $\frac{12}{20}$ is equivalent to $\frac{3}{5}$.

Rule

> To form a fraction equivalent to a given fraction, multiply (or divide) both the numerator and denominator by the same number.

Example 1 Write a fraction with a denominator of 42 that is equivalent to $\frac{5}{6}$.

$\frac{5}{6} = \frac{?}{42}$



Since the denominator is multiplied by 7, the numerator must also be multiplied by 7.

The fraction $\frac{35}{42}$ is equivalent to $\frac{5}{6}$.

Example 2 Write a fraction with a numerator of 4 that is equivalent to $\frac{24}{54}$.

$\frac{24}{54} = \frac{4}{?}$

$\frac{24}{54} \overset{\div 6}{=} \frac{4}{9}$

Since the numerator is divided by 6, the denominator must also be divided by 6.

The fraction $\frac{4}{9}$ is equivalent to $\frac{24}{54}$.

EXERCISE 5

1. Form equivalent fractions by completing the following.

(a) $\frac{5}{6} = \frac{\quad}{12}$ (b) $\frac{2}{7} = \frac{10}{\quad}$

(c) $\frac{13}{26} = \frac{\quad}{2}$ (d) $\frac{\quad}{5} = \frac{12}{30}$

(e) $\frac{9}{\quad} = \frac{27}{33}$ (f) $\frac{42}{100} = \frac{21}{\quad}$

(g) $\frac{36}{20} = \frac{\quad}{5}$ (h) $\frac{\quad}{28} = \frac{5}{7}$

2. Write a fraction with a denominator of 42 that is equivalent to each of the following.

(a) $\frac{9}{6} =$ (b) $\frac{11}{14} =$ (c) $\frac{12}{21} =$

3. Write a fraction with a denominator of 60 that is equivalent to each of the following.

(a) $\frac{1}{5} =$ (b) $\frac{7}{10} =$ (c) $\frac{8}{15} =$

ADDING AND SUBTRACTING FRACTIONS WITH THE SAME DENOMINATOR

To add fractions that have the same denominator, place the sum of the numerators over the common denominator. For subtraction, place the difference of the numerators over the common denominator.

Example 1

$$\frac{12}{8} + \frac{7}{8}$$

$$= \frac{12 + 7}{8}$$

$$= \frac{19}{8}$$

$$= 2\frac{3}{8}$$

Example 2

$$\frac{7}{9} - \frac{4}{9}$$

$$= \frac{7 - 4}{9}$$

$$= \frac{3}{9}$$

$$= \frac{1}{3}$$

ADDING AND SUBTRACTING FRACTIONS WITH DIFFERENT DENOMINATORS

Fractions cannot be added or subtracted unless they have the same denominator. If the denominators are different, you must find a common denominator.

Rule

To add or subtract fractions that have different denominators:

(a) - calculate the LCM of the denominators,

(b) - use the LCM to write equivalent fractions for each of the given fractions, and

(c) - add or subtract as indicated.

Example 1 Add: $\frac{1}{8} + \frac{5}{6}$

(a) - Find the LCM of the denominators.

$8 = 2 \times 2 \times 2$
$6 = 2 \times 3$

$LCM = 2 \times 2 \times 2 \times 3$
$= 24$

(b) - Use 24 as the denominator to write equivalent fractions for $\frac{1}{8}$ and $\frac{5}{6}$.

$\frac{1}{8} = \frac{3}{24}$ (×3 / ×3)

$\frac{5}{6} = \frac{20}{24}$ (×4 / ×4)

(c) - Add the fractions.

$$\frac{1}{8} + \frac{5}{6}$$

$$= \frac{3}{24} + \frac{20}{24}$$

$$= \frac{23}{24}$$

The sum of $\frac{1}{8}$ and $\frac{5}{6}$ is $\frac{23}{24}$.

Example 2 Subtract: $\frac{20}{35} - \frac{20}{45}$

$$\frac{20}{35} - \frac{20}{45}$$

$$= \frac{180}{315} - \frac{140}{315}$$

$$= \frac{40}{315}$$

$$= \frac{8}{63}$$

Computation

$35 = 5 \times 7$
$45 = 3 \times 3 \times 5$

$LCM = 3 \times 3 \times 5 \times 7$
$= 315$

$\frac{20}{35} = \frac{180}{315}$ (×9 / ×9)

$\frac{20}{45} = \frac{140}{315}$ (×7 / ×7)

EXERCISE 6 Perform the following operations. Show how you obtained your LCM in the computation column. Write your answer in lowest terms.

	Computation
1. $\frac{3}{4} + \frac{9}{10}$	
2. $\frac{15}{8} - \frac{11}{12}$	
3. $\frac{1}{6} + \frac{9}{40}$	
4. $\frac{41}{60} - \frac{20}{36}$	
5. $\frac{56}{144} + \frac{3}{24} + \frac{1}{2}$	

ADDING MIXED NUMBERS

Recall that a mixed number is made up of a natural number plus a fraction. To add mixed numbers, add the natural numbers separately from the fractions. The answer is the combined sum of the natural numbers and fractions.

Remember

> The fractions of a mixed number cannot be added unless they have the same denominator.

Example 1 Add $2\frac{1}{4}$ and $5\frac{2}{3}$.

$$\begin{array}{rcl} 2\frac{1}{4} & = & 2\frac{3}{12} \\ +5\frac{2}{3} & = & +5\frac{8}{12} \\ \hline & & 7\frac{11}{12} \end{array}$$

Notice that the fractions were changed to the same denominator before they were added.

$\frac{3}{12} + \frac{8}{12} = \frac{11}{12}$

$2 + 5 = 7$

The sum of $2\frac{1}{4}$ and $5\frac{2}{3}$ is $7\frac{11}{12}$.

Example 2 Add $36\frac{3}{4}$ and $17\frac{5}{6}$.

$$\begin{array}{rcl} 36\frac{3}{4} & = & 36\frac{9}{12} \\ +17\frac{5}{6} & = & +17\frac{10}{12} \\ \hline & & 53\frac{19}{12} \end{array} \quad = \quad 53 + 1\frac{7}{12} \quad = \quad 54\frac{7}{12}$$

Notice that $53\frac{19}{12}$ is a natural number plus an improper fraction. Change $\frac{19}{12}$ to a mixed number; $\frac{19}{12}$ equals $1\frac{7}{12}$. Add the natural numbers, 53 and 1, to obtain the final answer of $54\frac{7}{12}$.

EXERCISE 7 Add the following mixed numbers. Be sure to change to common denominators first. Write your answers in lowest terms.

1. $1\frac{3}{4} + 5\frac{1}{2}$

2. $4\frac{3}{4} + 3\frac{3}{5}$

3. $5\frac{3}{8} + 7\frac{3}{4}$

4. $3\frac{4}{5} + 6\frac{1}{3}$

5. $6\frac{2}{5} + 5\frac{3}{10}$

6. $2\frac{4}{7} + 5\frac{5}{14}$

7. $4\frac{5}{6} + 2\frac{5}{12}$

8. $8\frac{5}{16} + 2\frac{3}{8}$

SUBTRACTING MIXED NUMBERS

To subtract mixed numbers, you use the same approach used in adding mixed numbers.

Example 1 Subtract $4\frac{3}{7}$ from $6\frac{2}{3}$.

$$\begin{array}{rcl} 6\frac{2}{3} & = & 6\frac{14}{21} \\ -4\frac{3}{7} & = & -4\frac{9}{21} \\ \hline & & 2\frac{5}{21} \end{array}$$

$\frac{14}{21} - \frac{9}{21} = \frac{5}{21}$

$6 - 4 = 2$

Example 2 Subtract $8\frac{3}{4}$ from $14\frac{5}{12}$.

$$\begin{array}{rcl} 14\frac{5}{12} & = & 14\frac{5}{12} \\ -8\frac{3}{4} & = & -8\frac{9}{12} \end{array}$$

$\frac{9}{12}$ cannot be subtracted from a smaller fraction. To make the first fraction, $\frac{5}{12}$, larger follow this borrowing process:

$$\begin{aligned} 14\frac{5}{12} &= 14 + \frac{5}{12} \\ &= 13 + 1 + \frac{5}{12} \\ &= 13 + \frac{12}{12} + \frac{5}{12} \\ &= 13 + \frac{17}{12} \end{aligned}$$

Note: $1 = \frac{1}{1}$

$\frac{1}{1} = \frac{12}{12}$

Now, the subtraction can be completed.

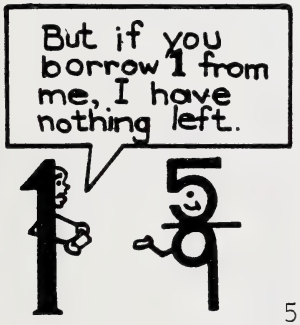
$$\begin{array}{rcl} 14\frac{5}{12} & = & 14\frac{5}{12} = 13\frac{17}{12} \\ -8\frac{3}{4} & = & -8\frac{9}{12} = -8\frac{9}{12} \\ \hline & & 5\frac{8}{12} = 5\frac{2}{3} \end{array}$$

EXERCISE 8 Subtract the following mixed numbers. Be sure to change to common denominators first. Write your answers in lowest terms.

1. $5\frac{4}{5}$
$-1\frac{7}{10}$

2. $4\frac{2}{3}$
$-1\frac{5}{6}$

3. $1\frac{1}{4}$
$-\frac{5}{6}$



4. $8\frac{11}{12}$
$-3\frac{1}{4}$

5. $9\frac{5}{6}$
$-2\frac{3}{8}$

6. $3\frac{7}{10}$
$-1\frac{2}{3}$

7. $6\frac{3}{4}$
$-4\frac{9}{10}$

8. $7\frac{5}{7}$
$-4\frac{13}{14}$

APPLICATIONS

EXERCISE 9 Solve the following word problems, using all of these steps.

1. Tell what the variable represents.
2. Write an equation using the variable.
3. Show each step used in solving the equation and your computation.
4. Write a statement answering the question asked.

1. Marge was making pies. She had $7\frac{1}{3}$ cups of flour in her flour canister. She used $5\frac{3}{4}$ cups of flour in her pastry. How much flour is left in the canister?

variable

equation

Computation

statement

2. Marge took 8 of her pies to a large family reunion. After everyone ate, there were $2\frac{5}{6}$ pies left. How many pies did the family eat?

Computation

3. Albert, Slim and Whip ran in a relay race. Albert ran $1\frac{1}{3}$ laps, Slim ran $2\frac{2}{3}$ laps and Whip ran the last $3\frac{1}{5}$ laps. What is the total number of laps that the boys ran?

Computation

4. Joey opened a can of dog food to feed his dogs. He gave Ralph $\frac{1}{4}$ of the can and Fred $\frac{1}{3}$ of the can. How much dog food is left in the can?

Computation

KEY TO SELF-CORRECTING EXERCISES IN SECTION 6

SELF-CORRECTING EXERCISE A, page 9.

1. $4 = 2 \times 2$
   $6 = 2 \times 3$

   $LCM = 2 \times 2 \times 3$
   $= 12$

In the largest group of each factor, 2 occurs twice and 3 occurs once.

2. $8 = 2 \times 2 \times 2$
   $24 = 2 \times 2 \times 2 \times 3$

   $LCM = 2 \times 2 \times 2 \times 3$
   $= 24$

In the largest group of each factor, 2 occurs three times and 3 occurs once.

3. $5 = 5$
   $9 = 3 \times 3$

   $LCM = 3 \times 3 \times 5$
   $= 45$

In the largest group of each factor, 3 occurs twice and 5 occurs once.

4. $3 = 3$
   $6 = 2 \times 3$
   $10 = 2 \times 5$

   $LCM = 2 \times 3 \times 5$
   $= 30$

In the largest group of each factor, 2 occurs once, 3 occurs once and 5 occurs once.

5. $42 = 2 \times 3 \times 7$
$90 = 2 \times 3 \times 3 \times 5$

$LCM = 2 \times 3 \times 3 \times 5 \times 7$
$= 630$

In the largest group of each factor, 2 occurs once, 3 occurs twice, 5 occurs once and 7 occurs once.

6. $105 = 3 \times 5 \times 7$
$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$
$= 2520$

In the largest group of each factor, 2 occurs three times, 3 occurs twice, 5 occurs once and 7 occurs once.

Key to the Exercises in Section 6

Exercise 1

1. (a) $5 > -5$ (b) $0 > -2$

   (c) $-12 < -3$ (d) $-32 < 7$

2. (a) $-6+8=+2$ (b) $-9+14=5$

   (c) $8--14=8++14=22$ (d) $-21--7=-21++7=-14$

3. (a) $3^4 = 3\times3\times3\times3=81$ (b) $7^3 = 7\times7\times7=343$

   (c) $10^5 = 10\times10\times10\times10\times10=100\,000$ (d) $13^2 = 13\times13=169$

4. $2^5 \times 2^2 \times 2^1 = 2^{5+2+1} = 2^8$
   $= 2\times2\times2\times2\times2\times2\times2\times2=256$

Exercise 2

1. $\frac{10}{65}=\frac{2}{13}$ 2. $\frac{16}{64}=\frac{1}{4}$

3. $\frac{24}{36}=\frac{2}{3}$ 4. $\frac{8}{48}=\frac{1}{6}$

5. $\frac{16}{32}=\frac{1}{2}$ 6. $\frac{22}{121}=\frac{2}{11}$

7. $\frac{14}{56}=\frac{1}{4}$ 8. $\frac{48}{52}=\frac{12}{13}$

9. $\frac{30}{55}=\frac{6}{11}$ 10. $\frac{20}{110}=\frac{2}{11}$

11. $\frac{117}{130}=\frac{9}{10}$ 12. $\frac{99}{132}=\frac{3}{4}$

Exercise 3

1. (a) $\frac{22}{7} = 3\frac{1}{7}$ (b) $\frac{37}{10} = 3\frac{7}{10}$

(c) $\frac{17}{5} = 3\frac{2}{5}$ (d) $\frac{23}{8} = 2\frac{7}{8}$

(e) $\frac{26}{12} = 2\frac{2}{12} = 2\frac{1}{6}$ (f) $\frac{28}{8} = 3\frac{4}{8} = 3\frac{1}{2}$

Exercise 4

1. $\frac{14}{63} = \frac{2 \times \overset{1}{\cancel{7}}}{3 \times 3 \times \underset{1}{\cancel{7}}} = \frac{2}{9}$

2. $\frac{10}{105} = \frac{2 \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \times 3 \times 7} = \frac{2}{21}$

3. $\frac{64}{256} = \frac{\overset{1}{\cancel{8}} \times \overset{1}{\cancel{8}}}{\underset{1}{\cancel{8}} \times \underset{1}{\cancel{8}} \times 4} = \frac{1}{4}$

4. $\frac{34}{51} = \frac{2 \times \overset{1}{\cancel{17}}}{3 \times \underset{1}{\cancel{17}}} = \frac{2}{3}$

Exercise 5

1. (a) $\frac{5}{6} = \frac{10}{12}$ (b) $\frac{2}{7} = \frac{10}{35}$ (c) $\frac{13}{26} = \frac{1}{2}$ (d) $\frac{2}{5} = \frac{12}{60}$

(e) $\frac{9}{11} = \frac{27}{33}$ (f) $\frac{42}{100} = \frac{21}{50}$ (g) $\frac{36}{20} = \frac{9}{5}$ (h) $\frac{20}{28} = \frac{5}{7}$

2. (a) $\frac{9}{6} = \frac{63}{42}$ (b) $\frac{11}{14} = \frac{33}{42}$ (c) $\frac{12}{21} = \frac{24}{42}$

3. (a) $\frac{1}{5} = \frac{12}{60}$ (b) $\frac{7}{10} = \frac{42}{60}$ (c) $\frac{8}{15} = \frac{32}{60}$

Exercise 6

1. $\frac{3}{4}+\frac{9}{10}$
$=\frac{15}{20}+\frac{18}{20}$
$=\frac{33}{20}$
$=1\frac{13}{20}$

2. $\frac{15}{8}-\frac{11}{12}$
$=\frac{45}{24}-\frac{22}{24}$
$=\frac{23}{24}$

3. $\frac{1}{6}+\frac{9}{40}$
$=\frac{20}{120}+\frac{27}{120}$
$=\frac{47}{120}$

4. $\frac{41}{60}-\frac{20}{36}=\frac{123}{180}-\frac{100}{180}=\frac{23}{180}$

5. $\frac{56}{144}+\frac{3}{24}+\frac{1}{2}=\frac{56}{144}+\frac{18}{144}+\frac{72}{144}$
$=\frac{146}{144}$
$=1\frac{2}{144}=1\frac{1}{72}$

Exercise 7

1. $1\frac{3}{4}=1\frac{3}{4}$
$+\ 5\frac{1}{2}=5\frac{2}{4}$
$6\frac{5}{4}=7\frac{1}{4}$

2. $4\frac{3}{4}=4\frac{15}{20}$
$+\ 3\frac{3}{5}=3\frac{12}{20}$
$7\frac{27}{20}=8\frac{7}{20}$

3. $5\frac{3}{8}=5\frac{3}{8}$
$+\ 7\frac{3}{4}=7\frac{6}{8}$
$12\frac{9}{8}=13\frac{1}{8}$

4. $3\frac{4}{5}=3\frac{12}{15}$
$+\ 6\frac{1}{3}=6\frac{5}{15}$
$9\frac{17}{15}=10\frac{2}{15}$

5. $6\frac{2}{5} = 6\frac{4}{10}$
$+ 5\frac{3}{10} = 5\frac{3}{10}$
$11\frac{7}{10}$

6. $2\frac{4}{7} = 2\frac{8}{14}$
$+ 5\frac{5}{14} = 5\frac{5}{14}$
$7\frac{13}{14}$

7. $4\frac{5}{6} = 4\frac{10}{12}$
$+ 2\frac{5}{12} = 2\frac{5}{12}$
$6\frac{15}{12} = 7\frac{3}{12} = 7\frac{1}{4}$

8. $8\frac{5}{16} = 8\frac{5}{16}$
$+ 2\frac{3}{8} = 2\frac{6}{16}$
$10\frac{11}{16}$

Exercise 8

1. $5\frac{4}{5} = 5\frac{8}{10}$
$- 1\frac{7}{10} = -1\frac{7}{10}$
$4\frac{1}{10}$

2. $4\frac{2}{3} = 4\frac{4}{6} = 3\frac{10}{6}$
$- 1\frac{5}{6} = -1\frac{5}{6} = -1\frac{5}{6}$
$2\frac{5}{6}$

3. $1\frac{1}{4} = 1\frac{6}{24} = \frac{30}{24}$
$- \frac{5}{6} = -\frac{20}{24} = \frac{20}{24}$
$\frac{10}{24} = \frac{5}{12}$

4. $8\frac{11}{12} = 8\frac{11}{12}$
$- 3\frac{1}{4} = 3\frac{3}{12}$
$5\frac{8}{12} = 5\frac{2}{3}$

5. $9\frac{5}{6} = 9\frac{20}{24}$
$- 2\frac{3}{8} = -2\frac{9}{24}$
$7\frac{11}{24}$

6. $3\frac{7}{10} = 3\frac{21}{30}$
$- 1\frac{2}{3} = -1\frac{20}{30}$
$2\frac{1}{30}$

7. $6\frac{3}{4} = 6\frac{15}{20} = 5\frac{35}{20}$

$-4\frac{9}{10} = -4\frac{18}{20} = 4\frac{18}{20}$

$= 1\frac{17}{20}$

8. $7\frac{5}{7} = 7\frac{10}{14} = 6\frac{24}{24}$

$-4\frac{13}{14} = 4\frac{13}{14} = 4\frac{13}{24}$

$= 2\frac{11}{24}$

Exercise 9

1. $x = 7\frac{1}{3} - 5\frac{3}{4}$

$= 1\frac{7}{12}$

$7\frac{1}{3} = 7\frac{4}{12} = 6\frac{16}{12}$

$-5\frac{3}{4} = 5\frac{9}{12} = 5\frac{9}{12}$

$= 1\frac{7}{12}$

2. $x = 8 - 2\frac{5}{6}$

$= 7\frac{6}{6} - 2\frac{5}{6}$

$= 5\frac{1}{6}$

3. $x = 1\frac{1}{3} + 2\frac{2}{3} + 3\frac{1}{5}$

$= 1\frac{5}{15} + 2\frac{10}{15} + 3\frac{3}{15}$

$= 6\frac{18}{15}$

$= 7\frac{3}{15} = 7\frac{1}{5}$

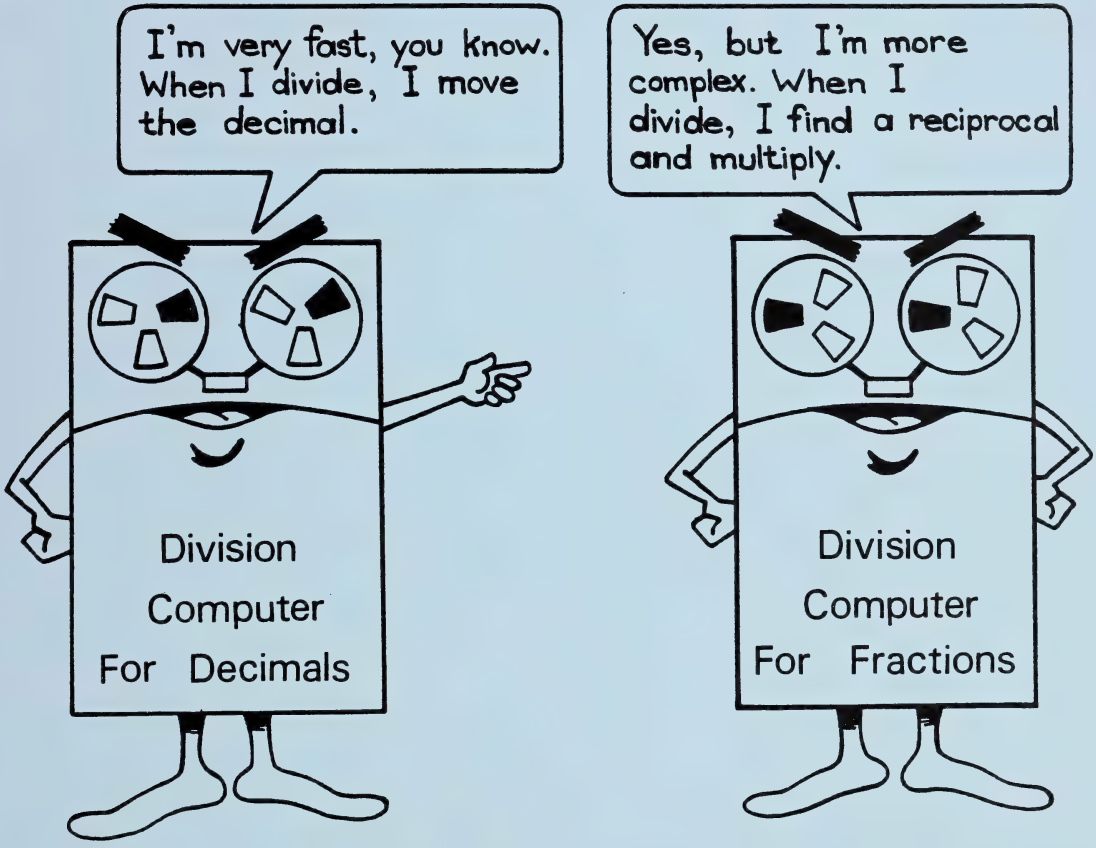
4. $x = 1 - \left(\frac{1}{4} + \frac{1}{3}\right)$

$= \frac{12}{12} - \left(\frac{3}{12} + \frac{4}{12}\right)$

$= \frac{12}{12} - \frac{7}{12}$

$= \frac{5}{12}$

FRACTIONS AND DECIMALS


I'm very fast, you know. When I divide, I move the decimal.
Division Computer For Decimals
Yes, but I'm more complex. When I divide, I find a reciprocal and multiply.
Division Computer For Fractions

In this section you will learn about:

- multiplying fractions and mixed numbers
- dividing fractions and mixed numbers
- adding, subtracting, multiplying and dividing decimal numbers
- using short-cut rules for multiplying and dividing by powers of 10

KEEPING SKILLFUL

EXERCISE 1

1. Write in words how the following would be read.

(a) 6^5 _____

(b) 24^2 _____

(c) 18^3 _____

2. Fill in the blanks with the missing power.

(a) $6^8 \times$ _____ $= 6^{11}$ (b) $10^7 \div$ _____ $= 10^1$

(c) _____ $\times 12 = 12^4$ (d) _____ $\div 3^2 = 3^0$

3. Fill in the blanks with the missing integer.

(a) $6 +$ _____ $= -2$ (b) _____ $- 8 = -6$

(c) $+4 -$ _____ $= 10$ (d) $-11 -$ _____ $= 3$

4. Perform the indicated operations. Write your answers in lowest terms.

(a) $\frac{3}{8} + \frac{5}{7}$ (b) $\frac{23}{40} - \frac{3}{16}$

MULTIPLYING FRACTIONS

To multiply fractions, multiply their numerators, then multiply their denominators.

Example 1 Multiply: $\frac{1}{10} \times \frac{6}{9}$

Method 1:

$$\frac{1}{10} \times \frac{6}{9} = \frac{1 \times 6}{10 \times 9}$$

Multiply the numerators, then multiply the denominators.

$$= \frac{6}{90}$$

$$= \frac{\cancel{6}^{\,2}}{\cancel{90}_{\,15}}$$

Reduce the answer to the lowest terms.

$$= \frac{1}{15}$$

In Method 1, the fraction was reduced after the multiplication was done. However, it is often easier to do the reducing (simplifying) before doing the multiplication.

To reduce fractions that are being multiplied, reduce any numerator with any denominator, by dividing by a common factor. Method 2 shows how fractions can be simplified before they are multiplied.

Method 2:

$$\frac{1}{10} \times \frac{6}{9} = \frac{1 \times \cancel{6}^{\,2}}{\cancel{10}_{\,5} \times \cancel{9}^{\,3}}$$

3 divides into both 6 and 9.

2 divides into both 2 and 10.

$$= \frac{1 \times 1}{5 \times 3}$$

$$= \frac{1}{15}$$

Both Method 1 and 2 give a final product of $\frac{1}{15}$.

Example 2 Multiply: $\frac{5}{8} \times \frac{10}{11}$

$$\frac{5}{8} \times \frac{10}{11} = \frac{5 \times \overset{5}{\cancel{10}}}{\underset{4}{\cancel{8}} \times 11}$$ 2 divides into both 10 and 8.

$$= \frac{25}{44}$$

Warning In Example 2, you cannot reduce the remaining two 5's that are in the numerator. *You can reduce a numerator only with a denominator.*

EXERCISE 2 Find the following products by simplifying before you multiply.

1. $\frac{7}{36} \times \frac{6}{10}$

2. $\frac{4}{18} \times \frac{8}{9}$

3. $\frac{25}{30} \times \frac{12}{19}$

4. $\frac{7}{49} \times \frac{33}{121}$

CHANGING MIXED NUMBERS TO IMPROPER FRACTIONS

Recall that:

- a mixed number is a natural number plus a fraction,
- an improper fraction has a numerator greater than or equal to its denominator.

On page 5 of Lesson 5 you learned that an improper fraction is changed to a mixed number by dividing its denominator into its numerator.

$\frac{17}{7} = 2\frac{3}{7}$

$7\overline{)17}$ quotient 2 ← This is the natural number part.

$\frac{14}{3}$ ← The remainder over the divisor is the fractional part.

Now you will learn the opposite procedure.

Rule

> To change a mixed number to an improper fraction, multiply the denominator by the natural number, then add on the numerator.

$2\frac{3}{7} = \frac{(7\times2)+3}{7} = \frac{17}{7}$

SELF-CORRECTING EXERCISE A

1. Change each improper fraction to a mixed number in lowest terms.

(a) $\frac{15}{7}$ _____ (b) $\frac{23}{10}$ _____ (c) $\frac{14}{5}$ _____

(d) $\frac{31}{10}$ _____ (e) $\frac{22}{12}$ _____ (f) $\frac{24}{16}$ _____

2. Change each mixed number to an improper fraction.

(a) $1\frac{1}{10}$ _____ (b) $2\frac{3}{8}$ _____ (c) $1\frac{11}{12}$ _____

(d) $5\frac{4}{5}$ (e) $6\frac{1}{4}$ (f) $2\frac{1}{3}$

Check the answers with those on page 21 of this section.
Correct any errors in your work.

MULTIPLYING MIXED NUMBERS

To multiply two fractions, you must multiply the numerators and the denominators of the fractions. Before multiplying mixed numbers, first change them to fractions.

Rule

> To multiply mixed numbers, change them to improper fractions, then multiply.

Example Find the product of $3\frac{1}{2}$ and $1\frac{2}{7}$.

$$3\frac{1}{2} \times 1\frac{2}{7} = \frac{7}{2} \times \frac{9}{7}$$ ← Change the mixed numbers to improper fractions.

$$= \frac{\cancel{7}^{1} \times 9}{2 \times \cancel{7}_{1}}$$ ← Simplify before you multiply.

$$= \frac{9}{2}$$

$$= 4\frac{1}{2}$$

EXERCISE 3 Find the following products by simplifying before you multiply. Write your answers in lowest terms.

1. $\frac{7}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{5}$

$= \frac{7}{10}$

2. $\frac{2}{3} \times 1\frac{1}{4}$

3. $\frac{5}{6} \times \frac{9}{10}$

4. $\frac{3}{5} \times \frac{4}{9}$

5. $\frac{5}{6} \times 1\frac{1}{5}$

$= \frac{\cancel{5}^{1}}{\cancel{6}_{1}} \times \frac{\cancel{6}^{1}}{\cancel{5}_{1}}$

$= \frac{1}{1}$

$= 1$

6. $\frac{3}{10} \times 2\frac{2}{9}$

7. $\frac{15}{16} \times 1\frac{3}{5}$

8. $2\frac{6}{7} \times \frac{14}{15}$

9. $1\frac{3}{4} \times 1\frac{5}{7}$

10. $1\frac{11}{16} \times 3\frac{1}{5}$

11. $9\frac{4}{5} \times 3\frac{3}{14}$

12. $6\frac{2}{3} \times 2\frac{7}{16}$

RECIPROCALS

If $a \times b = 1$ each of the numbers **a** and **b** is the reciprocal of the other.

Example 1 What is the reciprocal of $\frac{5}{3}$?

$$\frac{5}{3} \times \frac{3}{5} = \frac{\cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} = \frac{1}{1} \text{ or } 1$$

Since the product of $\frac{5}{3}$ and $\frac{3}{5}$ is 1, the reciprocal of $\frac{5}{3}$ is $\frac{3}{5}$.

The reciprocal of a fraction can be obtained by interchanging the numerator and denominator.

Rule

> To write a whole number as an improper fraction, put it over a denominator of 1.

Example 2 What is the reciprocal of 9?

Since 9 equals $\frac{9}{1}$, the reciprocal is $\frac{1}{9}$.

Example 3 What is the reciprocal of $4\frac{2}{5}$?

Since $4\frac{2}{5}$ equals $\frac{22}{5}$ the reciprocal is $\frac{5}{22}$.

SELF-CORRECTING EXERCISE B

Write the reciprocals of the following.

1. $\frac{8}{11}$ _____ 2. $6\frac{1}{3}$ _____ 3. 32 _____

4. 1 _____ 5. $9\frac{3}{4}$ _____ 6. $\frac{1}{10}$ _____

Check the answers with those on page 21 of this section.
Correct any errors in your work.

DIVIDING FRACTIONS

Recall that $5 \div 6$ can be written as $\frac{5}{6}$. Also, whole numbers can be written as improper fractions by putting them over a denominator of 1.

Follow this pattern:

$$5 \div 6 = \frac{5}{6}$$

$$\frac{5}{1} \div \frac{6}{1} = \frac{5}{6}$$

$$\frac{5}{1} \times \frac{1}{6} = \frac{5}{6}$$

Do you see that these two equations have the same answer?

From this you should realize that dividing by $\frac{6}{1}$, is the same as multiplying by $\frac{1}{6}$.

Does this work for all fractions? Pick any two fractions, say, $\frac{2}{3}$ and $\frac{5}{7}$. Recall that:

$\frac{2}{3} \div \frac{5}{7} = ?$ is the same question as $\frac{5}{7} \times ? = \frac{2}{3}$.

Using the above method,

$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$.

Does this satisfy $\frac{5}{7} \times ? = \frac{2}{3}$?

$\frac{\cancel{5}^{1}}{\cancel{7}_{1}} \times \frac{\cancel{14}^{2}}{\cancel{15}_{3}} = \frac{2}{3}$ It does.

So the above method works for $\frac{2}{3}$ and $\frac{5}{7}$.

You can try any other fractions and you will find that multiplying by the reciprocal of the divisor gives the correct quotient.

Rule

To divide by a fraction multiply by its reciprocal.

Example 1 Find the quotient of $\frac{6}{10}$ and $\frac{4}{9}$.

$$\frac{6}{10} \div \frac{4}{9} = \frac{6}{10} \times \frac{9}{4}$$ ← Change division to multiplication. Use the reciprocal.

$$= \frac{\overset{3}{\cancel{6}} \times 9}{10 \times \cancel{4}_{2}}$$ ← Simplify before you multiply.

$$= \frac{27}{20}$$

$$= 1\frac{7}{20}$$

The quotient is $1\frac{7}{20}$

Remember *When you change division to multiplication you always change the <u>second</u> fraction (the divisor) to its reciprocal.*

Example 2 Find the quotient of $\frac{27}{8}$ and $\frac{9}{12}$.

$$\frac{27}{8} \div \frac{9}{12} = \frac{27}{8} \times \frac{12}{9}$$

This is the divisor.

$$= \frac{\overset{3}{\cancel{27}} \times \cancel{12}^{3}}{\cancel{8}_{2} \times \cancel{9}_{1}}$$

$$= \frac{9}{2}$$

$$= 4\frac{1}{2}$$

SELF-CORRECTING EXERCISE C

Find the following quotients. Write your answers in lowest terms.

1. $\frac{5}{6} \div \frac{1}{2} =$

2. $\frac{18}{36} \div \frac{4}{7} =$

3. $\frac{25}{49} \div \frac{12}{63} =$

4. $\frac{8}{30} \div \frac{30}{45} =$

Check the answers with those on page 21 of this section.
Correct any errors in your work.

DIVIDING MIXED NUMBERS

When dividing by a fraction, the same result can be obtained by multiplying by the reciprocal of the fraction. Before dividing by a mixed number, change it to a fraction.

Rule

> To divide mixed numbers, change them to improper fractions and then multiply by the reciprocal of the second fraction.

Example Find the quotient of $3\frac{4}{7}$ and $2\frac{1}{2}$.

$$3\frac{4}{7} \div 2\frac{1}{2} = \frac{25}{7} \div \frac{5}{2}$$ ← Change the mixed numbers to improper fractions.

$$= \frac{25}{7} \times \frac{2}{5}$$ ← Change division to multiplication by the reciprocal.

$$= \frac{\cancel{25}^{5} \times 2}{7 \times \cancel{5}_{1}}$$ ← Simplify before you multiply.

$$= \frac{10}{7} \text{ or } 1\frac{3}{7}$$

EXERCISE 4 Find the following quotients. Write your answers in lowest terms.

1. $\frac{2}{3} \div \frac{5}{6}$

$= \frac{2}{\cancel{3}_{1}} \times \frac{\cancel{6}^{2}}{5}$

$= \frac{4}{5}$

2. $1\frac{1}{3} \div 1\frac{3}{5}$

3. $\frac{5}{8} \div \frac{3}{4}$

4. $6\frac{2}{3} \div 2\frac{1}{2}$

5. $2\frac{1}{6} \div 1\frac{1}{18}$

6. $5\frac{1}{3} \div 10$

7. $1\frac{1}{5} \div 2\frac{1}{10}$

$= \frac{6}{5} \div \frac{21}{10}$

$= \frac{\cancel{6}^{2}}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{\cancel{21}_{7}}$

$= \frac{4}{7}$

8. $1\frac{5}{9} \div 4\frac{2}{3}$

9. $28 \div 1\frac{3}{5}$

10. $2\frac{9}{13} \div \frac{21}{26}$

11. $3\frac{3}{4} \div 7\frac{1}{2}$

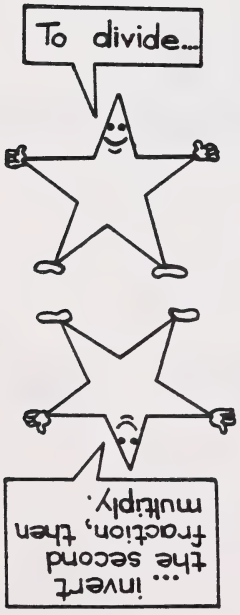
12. $5\frac{1}{2} \div 5\frac{1}{3}$

$= \frac{11}{2} \div \frac{16}{3}$

$= \frac{11}{2} \times \frac{3}{16}$

$= \frac{33}{32}$

$= 1\frac{1}{32}$



13. $4\frac{4}{5} \div 2\frac{1}{4}$

14. $5\frac{1}{4} \div 1\frac{1}{12}$

ADDING AND SUBTRACTING DECIMALS

Decimal numbers can always be added or subtracted by renaming them in expanded form. This is illustrated by the example below.

EXAMPLE

$$3.9 + 5.64 = 3 + \frac{9}{10} + 5 + \frac{6}{10} + \frac{4}{100}$$

$$= 3 + 5 + \frac{9}{10} + \frac{6}{10} + \frac{4}{100}$$

$$= 8 + \frac{15}{10} + \frac{4}{100}$$

$$= 8 + \frac{10}{10} + \frac{5}{10} + \frac{4}{100}$$

$$= 9 + \frac{5}{10} + \frac{4}{100}$$

$$= 9.54$$

When adding decimal numbers, you must add the digits that have the same place value. This applies to subtraction also. The easiest way to arrange this is to line up the decimal points.

Rule

> Line up the decimal points, when adding or subtracting decimal numbers.

Remember that a decimal comes at the end of a whole number. Thus, 26 means 26. or 26.000, as well.

Example 1

Add: 17 + 16.4 + 9.02

$$\begin{array}{r} {\scriptstyle 2} \\ 17.00 \\ 16.40 \\ 9.02 \\ \hline 42.42 \end{array}$$

Example 2

Subtract: 6.4 - 3.26

$$\begin{array}{r} {\scriptstyle 3\,1} \\ 6.\cancel{4}0 \\ 3.26 \\ \hline 3.14 \end{array}$$

← This zero is is placeholder.

Notice that placeholder zeros were used in Examples 1 and 2. When there is no number in a certain place value you assume that the number is 0.

EXERCISE 5 Do the following computations using decimal numbers. (Be sure to rewrite in vertical form before you add or subtract.)

1. 3 + 0.07 + 0.58

2. 43.75 + 89.669 + 0.4

3. 8.3664 + 0.32 + 0.008

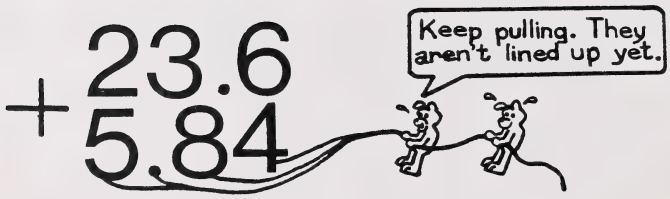
4. 10.8 + 0.5 + 12 + 0.761

5. 18.3 - 0.74

6. 7.663 - 0.87

7. 6 - 3.209

8. 17.329 - 9.309



MULTIPLYING DECIMAL NUMBERS

To multiply decimal numbers, multiply the numbers as you normally would. (Do not line up the decimal points.) Then follow this rule.

Rule

> The sum of the number of decimal places in the multipliers equals the number of decimal places in the product.

Example

$$\begin{array}{r} 2.04 \\ \times\ 5.3 \\ \hline 612 \\ 10\ 20 \\ \hline 10.812 \end{array}$$

2.04 ← 2 decimal places, plus one decimal place.
5.3 ←
10.812 ← 3 decimal places.

EXERCISE 6 Multiply the following decimal numbers. Use the rule above to place the decimal point after multiplying.

1. 7.39×1.8

2. 1.85×19

3. 8.601×4.3

4. 7.6×0.108

DIVIDING DECIMAL NUMBERS

Dividing by a Whole Number

When you divide a whole number into a decimal number, simply place a decimal point in the quotient directly above its position in the dividend.

If the dividend is a whole number, place a decimal point at the end of that whole number and another directly above it in the quotient.

Example 1

Divide 14 by 5

$$5\overline{)14.0}$$

$$\begin{array}{r} 2.8 \\ 5\overline{)14.0} \\ 10\times \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

← Extra zeros can be added after the decimal point.

Example 2

Divide 2.94 by 15

$$\begin{array}{r} 0.196 \\ 15\overline{)2.940} \\ 15\times\times \\ \hline 144 \\ 135 \\ \hline 90 \\ 90 \\ \hline 0 \end{array}$$

Dividing by a Decimal Number

If the divisor is a decimal number it must be changed to a whole number. To do this, move the decimal point of the divisor to the end of the number. *Then move the decimal point in the dividend the same number of places that it was moved in the divisor.*

The decimal point is then put in the quotient, directly above its new location in the dividend. The symbol ∧ (named the caret) shows the new location of the decimal point.

Example 3

Divide 33.4 by 0.25

$$0.25\overline{)33.40}$$

$$\begin{array}{r} 133.6 \\ 0.25\overline{)33.40} \\ 25\times\times\times \\ \hline 84 \\ 75 \\ \hline 90 \\ 75 \\ \hline 150 \\ 150 \\ \hline 0 \end{array}$$

Example 4

Divide 93 by 1.2

$$1.2\overline{)93.0}$$

$$\begin{array}{r} 77.5 \\ 1.2\overline{)93.0} \\ 84\times\times \\ \hline 90 \\ 84 \\ \hline 60 \\ 60 \\ \hline 0 \end{array}$$

EXERCISE 7

1. For the following, use the caret sign, $\wedge$, to show the new location of the decimal point in the divisor and dividend. *Do not* do the actual division. Question (a) is done for you. (Note: extra zeros may be added in the dividend.)

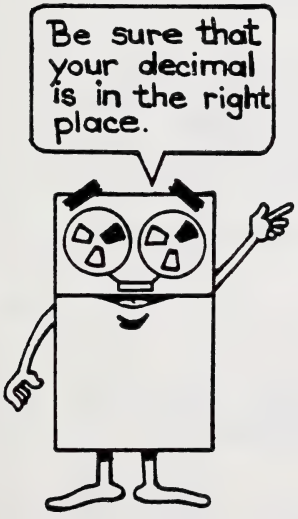
(a) $1.6\wedge\overline{)33.45\wedge}$ (b) $0.04\overline{)12.3}$

(c) $1.45\overline{)238}$ (d) $0.012\overline{)0.6}$

(e) $55\overline{)16.9}$ (f) $0.38\overline{)42.7}$

2. Find the quotient for each of the following. Continue dividing until you obtain a remainder of zero.

(a) $4\overline{)3.68}$ (b) $0.75\overline{)21}$



(c) $0.08\overline{)235.2}$ (d) $450\overline{)8.1}$

MULTIPLYING BY POWERS OF TEN

Look for a pattern in the following multiplication questions. You may wish to work out some of the products yourself, using the rules given on page 16 of this lesson.

$68.03 \times 10 = 680.3$
$68.03 \times 100 = 6803$
$68.03 \times 1000 = 68\ 030$

$68.03 \times 0.1 = 6.803$
$68.03 \times 0.01 = 0.6803$
$68.03 \times 0.001 = 0.068\ 03$

Short-cut Rule I

> When you multiply by 10, 100, 1000,..., move the decimal point the same number of places to the RIGHT as there are zeros in the power of ten.

Example 1 $68.03 \times 100\ 000 = ?$

Since 100 000 has 5 zeros, move the decimal 5 places to the right. $68.03 \times 100\ 000 = 6\ 803\ 000$

Short-cut Rule II

> When you multiply by 0.1, 0.01, 0.001, ..., move the decimal point the same number of places to the LEFT as the number of decimal places in the power of ten.

Example 2 $68.03 \times 0.000\ 01 = ?$

Since 0.000 01 has 5 decimal places, move the decimal 5 places to the left. $68.03 \times 0.000\ 01 = 0.000\ 680\ 3$

EXERCISE 8 Multiply the following using the above short-cut rules.

1. $0.03 \times 1000 =$ _____
2. $58.4 \times 0.01 =$ _____
3. $973.6 \times 0.0001 =$ _____
4. $15 \times 10 =$ _____
5. $0.81 \times 0.1 =$ _____
6. $2.344 \times 100 =$ _____
7. $237 \times 0.001 =$ _____
8. $7.002 \times 100\ 000 =$ _____

DIVIDING BY POWERS OF TEN

Look for a pattern in the following division questions. Again, you may want to work out some of the quotients for yourself by using long division.

981.25 ÷ 10 = 98.125	981.25 ÷ 0.1 = 9812.5
981.25 ÷ 100 = 9.8125	981.25 ÷ 0.01 = 98 125
981.25 ÷ 1000 = 0.981 25	981.25 ÷ 0.001 = 981 250

Short-cut Rule III

> When you divide by 10, 100, 1000, ... ,move the decimal point the same number of places to the LEFT as there are zeros in the power of ten.

Example 1 981.25 ÷ 10 000 = ?

Since 10 000 has 4 zeros, move the decimal 4 places to the left. 981.25 ÷ 10 000 = 0.098 125

Short-cut Rule IV

> When you divide by 0.1, 0.01, 0.001, ... ,move the decimal point the same number of places to the RIGHT as there are decimal places in the power of ten.

Example 2 981.25 ÷ 0.000 01 = ?

Since 0.000 01 has 5 decimal places, move the decimal 5 places to the right. 981.25 ÷ 0.000 01 = 98 125 000

EXERCISE 9 Divide the following using the above short-cut rules.

1. 1.8 ÷ 100 = _____
2. 0.023 ÷ 10 = _____
3. 2.04 ÷ 0.001 = _____
4. 28 ÷ 0.1 = _____
5. 3.9 ÷ 0.01 = _____
6. 50.1 ÷ 10 000 = _____
7. 434 ÷ 1000 = _____
8. 0.003 ÷ 0.01 = _____

KEY TO SELF-CORRECTING EXERCISES IN SECTION 7

SELF-CORRECTING EXERCISE A, page 5.

1. (a) $2\frac{1}{7}$ (b) $2\frac{3}{10}$ (c) $2\frac{4}{5}$

   (d) $3\frac{1}{10}$ (e) $1\frac{10}{12} = 1\frac{5}{6}$ (f) $1\frac{8}{16} = 1\frac{1}{2}$

   lowest terms

2. (a) $\frac{11}{10}$ (b) $\frac{19}{8}$ (c) $\frac{23}{12}$

   (d) $\frac{29}{5}$ (e) $\frac{25}{4}$ (f) $\frac{7}{3}$

SELF-CORRECTING EXERCISE B, page 8.

1. $\frac{11}{8}$ 2. $\frac{3}{19}$ 3. $\frac{1}{32}$

4. $\frac{1}{1} = 1$ 5. $\frac{4}{39}$ 6. 10

Note: zero has no reciprocal. No number times zero equals one.

SELF-CORRECTING EXERCISE C, page 11.

1. $\frac{5}{6} \div \frac{1}{2} = \frac{5}{6} \times \frac{2}{1}$

$= \frac{5 \times \cancel{2}^{1}}{\cancel{6}^{3} \times 1}$

$= \frac{5}{3}$ or $1\frac{2}{3}$

2. $\frac{18}{36} \div \frac{4}{7} = \frac{18}{36} \times \frac{7}{4}$

$= \frac{\cancel{18}^{1} \times 7}{\cancel{36}^{2} \times 4}$

$= \frac{7}{8}$

3. $\frac{25}{49} \div \frac{12}{63} = \frac{25}{49} \times \frac{63}{12}$

$= \frac{25}{\cancel{49}^{7}} \times \frac{\cancel{63}^{9^{3}}}{\cancel{12}^{4}}$

$= \frac{75}{28}$ or $2\frac{19}{28}$

4. $\frac{8}{30} \div \frac{30}{45} = \frac{8}{30} \times \frac{45}{30}$

$= \frac{\cancel{8}^{2\cancel{4}} \times \cancel{45}^{\cancel{9}^{1}}}{\cancel{30}^{\cancel{2}^{1}} \times \cancel{30}^{15^{5}}}$

$= \frac{2}{5}$

Key to the Exercises in Section 7

Exercise 1

1. (a) sixth to the fifth power

   (b) twenty-four squared or twenty-four to the second power

   (c) eighteen cubed or eighteen to the third power

2. (a) $6^8 \times 6^3 = 6^{11}$

   (b) $10^7 \div 10^6 = 10^1$

   (c) $12^3 \times 12 = 12^4$

   (d) $3^2 \div 3^2 = 3^0$

3. (a) $6 + -8 = -2$

   (b) $+2 - 8 = -6$

   (c) $+4 - -6 = 10$

   (d) $-11 - -14 = 3$

4. (a) $\frac{3}{8} + \frac{5}{7} = \frac{21}{56} + \frac{40}{56}$
   $= \frac{61}{56}$
   $= 1\frac{5}{56}$

   (b) $\frac{23}{40} - \frac{3}{16} = \frac{46}{80} - \frac{15}{80}$
   $= \frac{31}{80}$

Exercise 2

1. $\frac{7}{\cancel{36}_{6}} \times \frac{\overset{1}{\cancel{6}}}{10} = \frac{7}{60}$

2. $\frac{4}{\cancel{18}_{9}} \times \frac{\overset{4}{\cancel{8}}}{9} = \frac{16}{81}$

3. $\frac{\overset{5}{\cancel{25}}}{\cancel{30}_{\cancel{5}_{1}}} \times \frac{\overset{2}{\cancel{12}}}{19} = \frac{10}{19}$

4. $\frac{\overset{1}{\cancel{7}}}{\cancel{49}_{7}} \times \frac{\overset{3}{\cancel{33}}}{\cancel{121}_{11}} = \frac{3}{77}$

Exercise 3

1. $\frac{7}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{5} = \frac{7}{10}$

2. $\frac{2}{3} \times 1\frac{1}{4} = \frac{\cancel{2}^{1}}{3} \times \frac{5}{\cancel{4}_{2}} = \frac{5}{6}$

3. $\frac{\cancel{5}^{1}}{\cancel{6}_{2}} \times \frac{\cancel{9}^{3}}{\cancel{10}_{2}} = \frac{3}{4}$

4. $\frac{\cancel{3}^{1}}{5} \times \frac{4}{\cancel{9}_{3}} = \frac{14}{15}$

5. $\frac{5}{6} \times 1\frac{1}{5} = \frac{\cancel{5}^{1}}{\cancel{6}_{1}} \times \frac{\cancel{6}^{1}}{\cancel{5}_{1}} = 1$

6. $\frac{3}{10} \times 2\frac{2}{9} = \frac{\cancel{3}^{1}}{\cancel{10}_{1}} \times \frac{\cancel{20}^{2}}{9_{3}} = \frac{2}{3}$

7. $\frac{15}{16} \times 1\frac{3}{5} = \frac{\cancel{15}^{3}}{\cancel{16}_{2}} \times \frac{\cancel{8}^{1}}{\cancel{5}_{1}} = \frac{3}{2}$

8. $2\frac{6}{7} \times \frac{14}{15} = \frac{\cancel{20}^{4}}{\cancel{7}_{1}} \times \frac{\cancel{14}^{2}}{\cancel{15}_{3}} = \frac{8}{3}$

9. $1\frac{3}{4} \times 1\frac{5}{7} = \frac{\cancel{7}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{12}^{3}}{\cancel{7}_{1}} = 3$

10. $1\frac{11}{16} \times 3\frac{1}{5} = \frac{27}{\cancel{16}_{1}} \times \frac{\cancel{16}^{1}}{5} = \frac{27}{5}$

11. $9\frac{4}{5} \times 3\frac{3}{14} = \frac{\cancel{49}^{7}}{\cancel{5}_{1}} \times \frac{\cancel{45}^{9}}{\cancel{14}_{2}} = \frac{63}{2}$

12. $6\frac{2}{3} \times 2\frac{7}{16} = \frac{\cancel{20}^{5}}{\cancel{3}_{1}} \times \frac{\cancel{39}^{13}}{\cancel{16}_{4}} = \frac{65}{4}$

Exercise 4

2. $1\frac{1}{3} \div 1\frac{3}{5} = \frac{4}{3} \div \frac{8}{5} = \frac{\cancel{4}^{1}}{3} \times \frac{5}{\cancel{8}_{2}} = \frac{5}{6}$

3. $\frac{5}{8} \div \frac{3}{4} = \frac{5}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{3} = \frac{5}{6}$

4. $6\frac{2}{3} \div 2\frac{1}{2} = \frac{20}{3} \div \frac{5}{2} = \frac{\cancel{20}^{4}}{3} \times \frac{2}{\cancel{5}_{1}} = \frac{8}{3} = 2\frac{2}{3}$

5. $2\frac{1}{6} \div 1\frac{1}{18} = \frac{13}{6} \div \frac{19}{18} = \frac{13}{\cancel{6}_{1}} \times \frac{\cancel{18}^{3}}{19} = \frac{39}{19} = 2\frac{1}{19}$

6. $5\frac{1}{3} \div 10 = \frac{16}{3} \div \frac{10}{1} = \frac{\overset{8}{\cancel{16}}}{3} \times \frac{1}{\underset{5}{\cancel{10}}} = \frac{8}{15}$

7. $1\frac{1}{5} \div 2\frac{1}{10} = \frac{6}{5} \div \frac{21}{10} = \frac{6}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{10}}}{21} = \frac{12}{21}$

8. $1\frac{5}{9} \div 4\frac{2}{3} = \frac{14}{9} \div \frac{14}{3} = \frac{\overset{1}{\cancel{14}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{14}}} = \frac{1}{3}$

9. $28 \div 1\frac{3}{5} = \frac{28}{1} \div \frac{8}{5} = \frac{\overset{7}{\cancel{28}}}{1} \times \frac{5}{\underset{2}{\cancel{8}}} = \frac{35}{2} = 17\frac{1}{2}$

10. $2\frac{9}{13} \div \frac{21}{26} = \frac{35}{13} \div \frac{21}{26} = \frac{\overset{5}{\cancel{35}}}{\underset{1}{\cancel{13}}} \times \frac{\overset{2}{\cancel{26}}}{\underset{3}{\cancel{21}}} = \frac{10}{3} = 3\frac{1}{3}$

11. $3\frac{3}{4} \div 7\frac{1}{2} = \frac{15}{4} \div \frac{15}{2} = \frac{\overset{1}{\cancel{15}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{15}}} = \frac{1}{2}$

12. $5\frac{1}{2} \div 5\frac{1}{3} = \frac{11}{2} \div \frac{16}{3} = \frac{11}{2} \times \frac{3}{16} = \frac{33}{32} = 1\frac{1}{32}$

13. $4\frac{4}{5} \div 2\frac{1}{4} = \frac{24}{5} \div \frac{9}{4} = \frac{\overset{8}{\cancel{24}}}{5} \times \frac{4}{\underset{3}{\cancel{9}}} = \frac{32}{15} = 2\frac{2}{15}$

14. $5\frac{1}{4} \div 1\frac{1}{12} = \frac{21}{4} \div \frac{13}{12} = \frac{21}{\underset{1}{\cancel{4}}} \times \frac{\overset{3}{\cancel{12}}}{13} = \frac{63}{13} = 4\frac{11}{13}$

Exercise 5

1. 3.00
   0.07
   0.58
   3.65

2. 43.75
   89.669
   0.4
   133.819

3. 8.3664
   0.32
   0.008
   8.6944

4. 10.8
   0.5
   12.
   0.761
   24.061

5. 18.30
   − 0.74
   17.56

6. 7.663
   − 0.87
   6.793

7. 6.000
   − 3.209
   2.791

8. 17.329
   − 9.309
   8.020

Exercise 6

1. $7.39 \times 1.8 = 13.302$
2. $1.85 \times 19 = 35.15$
3. $8.601 \times 4.3 = 36.9843$
4. $0.108 \times 7.6 = 0.8208$

Exercise 7

1. (a) $1.6 \overset{\wedge}{)} \overset{\cdot}{33.4} \overset{\wedge}{5}$ (b) $0.04 \overset{\wedge}{)} \overset{\cdot}{12.30} \overset{\wedge}{}$ (c) $1.45 \overset{\wedge}{)} \overset{\cdot}{238.00} \overset{\wedge}{}$

   (d) $0.012 \overset{\wedge}{)} \overset{\cdot}{0.600} \overset{\wedge}{}$ (e) $55\overset{\cdot}{)}16.9$ (f) $0.38 \overset{\wedge}{)} 42.70 \overset{\wedge}{}$

2. (a) $4\overline{)3.68}$ $= 0.92$ (b) $0.75 \overset{\wedge}{)} 21.00 \overset{\wedge}{}$ $= 28.$

   (c) $0.08 \overset{\wedge}{)} 235.20 \overset{\wedge}{}$ $= 2940.$ (d) $450\overline{)8.100}$ $= 0.018$

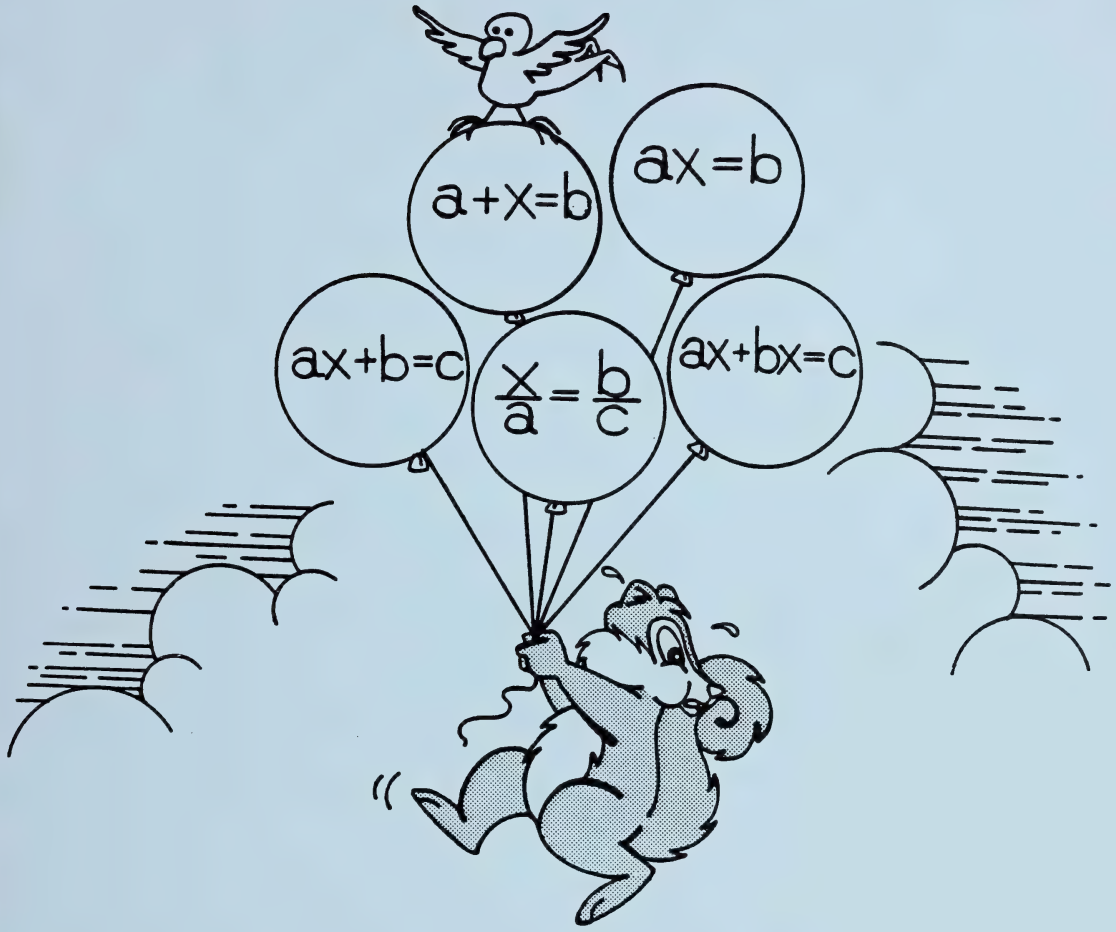
Exercise 8

1. $0.03 \times 1000 = 30$
2. $58.4 \times 0.01 = 0.584$
3. $973.6 \times 0.0001 = 0.09736$
4. $15 \times 10 = 150$
5. $0.81 \times 0.1 = 0.081$
6. $2.344 \times 100 = 234.4$
7. $237 \times 0.001 = 0.237$
8. $7.002 \times 100000 = 700200$

Exercise 9

1. $1.8 \div 100 = 0.018$
2. $0.023 \div 10 = 0.0023$
3. $2.04 \div 0.001 = 2040$
4. $28 \div 0.1 = 280$
5. $3.9 \div 0.01 = 390$
6. $50.1 \div 10000 = 0.00501$
7. $434 \div 1000 = 0.434$
8. $0.003 \div 0.01 = 0.3$

ALGEBRA



In this section you will learn about algebraic equations of the following types:

$a + x = b$

$ax = b$

$ax + b = c$

$\frac{x}{a} = \frac{b}{c}$

$ax + bx = c$

KEEPING SKILLFUL

EXERCISE 1

1. Use integers to represent the following expressions.

   (a) Five years in the future _____

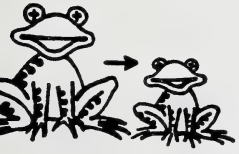
   (b) An expenditure of $12.00 _____

   (c) A bonus of $100.00 _____

   (d) 30 degrees below freezing _____

   (e) 15 m below sea level _____

2. List the following integers in order from the greatest to the least.



   (a) -15, +10, 0, +5, -7, -8

   _____

   (b) 0, -1, -7, -3, -2, -14

   _____

3. Add the following integers.

   (a) (+6) + (+3)

   (b) (-6) + (+3)

   (c) (-101) + (-49)

   (d) (+82) + (-143)

   (e) (+152) + (-77)

   (f) (-79) + (+79)

WHAT IS ALGEBRA?

In this lesson you will begin studying a new and exciting field of mathematics. Algebra is one of the easiest fields of mathematics that you will cover in this course. Algebra involves solving problems by translating an English sentence into a mathematical sentence or equation. In your previous grades, have you ever solved for an unknown (for n)? If you have done this, you are already well acquainted with algebra.

Solving Equations

In this lesson you will learn to solve the following types of algebraic equations:

$$a + x = b$$

$$ax = b$$

$$ax + b = c$$

$$\frac{x}{a} = \frac{b}{c}$$

$$ax + bx = c$$

In the above equations, x is the unknown and a, b, and c represent numbers.

Note

> Please remember that the left-hand side of an equation can be interchanged with the right-hand side without changing the meaning of the equation.

SOLVING EQUATIONS OF THE TYPE $a + x = b$

$$\underset{\text{A number}}{a} + \underset{\text{The unknown}}{x} = \underset{\text{A number}}{b}$$

To solve the above equation, you must first remember that addition and subtraction are inverse operations. To solve an addition equation, change it to a subtraction equation. To solve a subtraction equation, change it to an addition equation.

It is always a good idea to check your answer by substituting it into the original equation. If the left-hand side of the equation equals the right-hand side, your answer is correct.

Example 1 Solve the equation, $9 + x = 14$.

To solve for x you must remove the 9 that is added to the x. To do this, use the inverse operation.

$$9 + x = 14$$
$$x = 14 - 9 \quad \leftarrow \textit{Inverse operation.}$$
$$x = 5$$

The solution is $x = 5$.

Check: To check the above solution replace the x in the original equation with 5.

$$9 + x \overset{?}{=} 14 \quad \leftarrow \textit{Original equation.}$$
$$9 + 5 \mid 14 \quad \leftarrow \textit{Replace x with 5.}$$
$$14 \mid \checkmark$$

Notice that the left-hand side of the equation is equal to the right-hand side. The solution is, therefore, correct.

Example 2 Solve the equation, $-7 + x = 10$.

$$-7 + x = 10$$
$$x = 10 - (-7) \quad \text{Inverse operation.}$$
$$x = 10 + 7$$
$$x = 17$$

The solution is $x = 17$.

Check:

$$-7 + x \stackrel{?}{=} 10 \quad \text{Original equation.}$$
$$-7 + 17 \mid 10 \quad \text{Replace } x \text{ with } 17.$$
$$10 \mid \checkmark$$

The solution, $x = 17$, is correct.

SELF-CORRECTING EXERCISE A

Solve for x in each case. Be sure to check your solution.

1. $6 + x = 20$ Check:

2. $-11 + x = 4$ Check:

3. $x - 8 = 22$ Check:

Check the answers with those on page 20 of this section.
Correct any errors in your work.

EXERCISE 2 Solve for x in each case. Be sure to check your solution.

1. $17 + x = 35$ Check:

2. $-14 + x = -20$ Check:

3. $x - 13 = 4$ Check:

4. $1.6 + x = 5.7$ Check:

SOLVING EQUATIONS OF THE TYPE $ax=b$

$$ax = b$$

ax: The unknown multiplied by a number

b: A number

To solve the above equation, you must remember that ax means that the value of a is multiplied by the value of x. Also, recall that division is the inverse of multiplication. To solve a multiplication equation, change it to a division equation. To solve a division equation, change it to a multiplication equation.

Example 1 Solve the equation, $8x = 96$.

$$8x = 96$$
$$x = 96 \div 8 \quad \text{Inverse operation.}$$
$$x = 12$$

Check:

$$8x \stackrel{?}{=} 96 \quad \text{Original equation.}$$
$$8(12) \mid 96 \quad \text{Replace } x \text{ with } 12.$$
$$96 \mid \checkmark$$

The solution is $x = 12$.

Example 2 Solve the equation, $18x = 648$.

$$18 \times x = 648$$
$$x = 648 \div 18 \quad \text{Inverse operation.}$$
$$x = 36$$

Check:

$$18 \times x \stackrel{?}{=} 648 \quad \text{Original equation.}$$
$$18 \times 36 \mid 648 \quad \text{Replace } x \text{ with } 36.$$
$$648 \mid \checkmark$$

The solution is $x = 36$.

EXERCISE 3 Solve for x in each case. Be sure to check your solution.

1. $9x = 99$ Check:

2. $6 \times x = 96$ Check:

3. $7x = 4.9$ Check:

4. $144x = 1728$ Check:

SOLVING EQUATIONS OF THE TYPE **ax+b=c**

$$ax + b = c$$

- ax: The unknown multiplied by a number
- b: A number
- c: A number

Two examples of the above equation could be:

$7x + 12 = 33$ or $5x - 8 = 32$.

To solve for x you will be using the inverse operations applied to the previous equations of $a + x = b$ and $ax = b$.

Example 1 Solve the equation, $7x + 12 = 33$.

$7x + 12 = 33$ ← *Always do addition and subtraction first.*

$7x = 33 - 12$ ← *Inverse operation.*

$7x = 21$

$x = 21 \div 7$ ← *Second inverse operation.*

$x = 3$

Check:

$7x + 12 \stackrel{?}{=} 33$ ← *Original equation.*

$7(3) + 12 \mid 33$ ← *Replace x with 3.*

$21 + 12$

$33 \mid \checkmark$

The correct solution for $7x + 12 = 33$ is $x = 3$.

Example 2 Solve the equation, $5x - 8 = 32$.

$5x - 8 = 32$

$5x = 32 + 8$ ← *Inverse operation.*

$5x = 40$

$x = 40 \div 5$ ← *Second inverse operation.*

$x = 8$

$x = 8$ is the solution for $5x - 8 = 32$. On your own, check this solution by replacing x with 8.

SELF-CORRECTING EXERCISE B

Solve the following algebraic equations and check each solution.

1. $5x - 6 = 9$ Check:

2. $11x + 15 = 48$ Check:

3. $4x + 1.6 = 6.4$ Check:

Check the answers with those on page 20 of this section.
Correct any errors in your work.

EXERCISE 4 Solve for x in each case. Be sure to check your solution.

1. $8x - 9 = 63$ Check:

2. $8x + 11 = 27$ Check:

3. $14x - 28 = 28$ Check:

4. $1.2x + 5 = 125$ Check:

SOLVING EQUATIONS OF THE TYPE $ax+bx=c$

$$ax + bx = c$$

- ax: The unknown multiplied by a number.
- bx: The unknown multiplied by a number.
- c: A number

Two examples of the above equation could be:

$14x + 7x = 42$ or $9x - 3x = 54$.

To solve the above equations, it will be necessary to add or subtract the two quantities which contain the unknown and then solve for x.

Example 1 Solve the equation, $14x + 7x = 42$.

$14x + 7x = 42$ ← *Combine the two quantities which contain x.*

$21x = 42$

$x = 42 \div 21$ ← *Inverse operation.*

$x = 2$

Check:

$14x + 7x \stackrel{?}{=} 42$ ← *Original equation.*

$14(2) + 7(2) \mid 42$ ← *Replace x with 2.*

$28 + 14$

$42 \mid$ ✓

The solution is $x = 2$.

Example 2 Solve the equation, $9x - 3x = 54$.

$9x - 3x = 54$ ← *Subtract the two quantities which contain x.*

$6x = 54$

$x = 54 \div 6$ ← *Inverse operation.*

$x = 9$

The solution is $x = 9$.

On your own, check this solution by replacing x with 9.

SELF-CORRECTING EXERCISE C

Solve for x in each case. Be sure to check the solution.

1. $5x + 10x = 45$ Check:

2. $7x - 2x = 40$ Check:

3. $5.5x - 2.5x = 6$ Check:

Check the answers with those on page 21 of this section.
Correct any errors in your work.

The above equations were of the form $ax + bx = c$. You have also solved equations of the form $ax + b = c$ earlier in this lesson. Please remember that these two equations are not the same. The examples below illustrate this fact.

Form: $ax + bx = c$

$$2x + 5x = 14$$
$$7x = 14$$
$$x = 14 \div 2$$
$$x = 7$$

Form: $ax + b = c$

$$2x + 5 = 14$$
$$2x = 14 - 5$$
$$2x = 9$$
$$x = 4\frac{1}{2}$$

You can see that in the form $ax + bx = c$ the numbers containing the unknown x are added first while in the form $ax + b = c$ the number (b) is subtracted from the number (c).

EXERCISE 5 Solve for x in each case and be sure to check your solution.

1. $4x + 14x = 54$ Check:

2. $15x - x = 42$ Check:
(Hint: -x means the same as -1x)

3. $2.5x - 1.5x = 7$ Check:

4. $18x + 9x = 81$ Check:

SOLVING EQUATIONS OF THE TYPE $\frac{x}{a} = \frac{b}{c}$

The unknown → x A number → a $=$ b ← A number, c ← A number

The above equation can be related to working with fractions. If you replace a, b and c with numerals you can see that the equation involves working with equivalent fractions or with ratios. An example could be $\frac{x}{6} = \frac{30}{36}$.

Cross Multiplication

Cross multiplication will be used in solving for x in the equation $\frac{x}{a} = \frac{b}{c}$. Note how cross multiplication is done.

$\frac{x}{a} \times \frac{b}{c}$

$c \times x = b \times a$ ← *(c × x) and (b × a) mean cx and ba.*

or $cx = ba$

$x = ba \div c$ ← *Apply inverse operation to solve for x.*

or $x = \frac{b \times a}{c}$

Notice that in cross multiplication the denominator of one fraction is multiplied by the numerator of the other fraction and vice versa.

Example 1 Solve for x in $\frac{x}{6} = \frac{30}{36}$.

$$\frac{x}{6} = \frac{30}{36}$$

$$36x = 30 \times 6 \quad \longleftarrow \textit{Cross multiply.}$$

$$36x = 180$$

$$x = 180 \div 36 \quad \longleftarrow \textit{Inverse operation.}$$

$$x = 5$$

Check:

$$\frac{x}{6} \stackrel{?}{=} \frac{30}{36} \quad \longleftarrow \textit{Original equation.}$$

$$36x \mid 30 \times 6$$

$$36(5) \mid 180 \quad \longleftarrow \textit{Replace x with 5.}$$

$$180 \mid \checkmark$$

The solution is $x = 5$.

Example 2 Solve for x in $\frac{12}{6} = \frac{x}{7}$.

Note that equations of the type $\frac{x}{a} = \frac{b}{c}$ are the same as equations of the type $\frac{b}{c} = \frac{x}{a}$. Remember that the left-hand side of an equation can be interchanged with the right-hand side without changing the meaning of the equation.

$$\frac{12}{6} = \frac{x}{7}$$

$$6x = 12 \times 7 \quad \longleftarrow \textit{Cross multiply.}$$

$$6x = 84$$

$$x = 84 \div 6 \quad \longleftarrow \textit{Inverse operation.}$$

$$x = 14$$

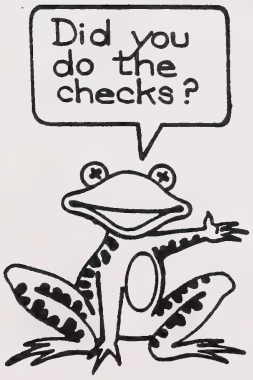
The solution is $x = 14$.

EXERCISE 6 Solve for x in each case and be sure to check your solution.

1. $\frac{x}{25} = \frac{25}{125}$ Check:

2. $\frac{x}{20} = \frac{42}{10}$ Check:

3. $\frac{30}{45} = \frac{x}{3}$ Check:



4. $\frac{6}{28} = \frac{x}{14}$ Check:

REVIEW

This section contains questions involving the five types of equations discussed in this lesson. Review the five methods again, then do the questions below.

EXERCISE 7 Solve for x in each case.

1. $-15 + x = 60$ Check:

2. $75x = 900$ Check:

3. $\frac{x}{3} = \frac{36}{12}$ Check:

4. $8x - 72 = 144$ Check:

5. $35x + 90x = 1250$ Check:

6. $5x - 3.2 = 6.8$ Check:

KEY TO SELF-CORRECTING EXERCISES IN SECTION 8

SELF-CORRECTING EXERCISE A, page 5.

1. $6 + x = 20$

$x = 20 - 6$

$x = 14$

Check: $6 + x \stackrel{?}{=} 20$

$6 + 14 \mid 20$

$20 \mid \checkmark$

2. $-11 + x = 4$

$x = 4 - (-11)$

$x = 4 + 11$

$x = 15$

Check: $-11 + x \stackrel{?}{=} 4$

$-11 + 15 \mid 4$

$4 \mid \checkmark$

3. $x - 8 = 22$

$x = 22 + 8$

$x = 30$

Check: $x - 8 \stackrel{?}{=} 22$

$30 - 8 \mid 22$

$22 \mid \checkmark$

SELF-CORRECTING EXERCISE B, page 10.

1. $5x - 6 = 9$

$5x = 9 + 6$

$5x = 15$

$x = 15 \div 5$

$x = 3$

Check: $5x - 6 \stackrel{?}{=} 9$

$5(3) - 6 \mid 9$

$15 - 6 \mid$

$9 \mid \checkmark$

2. $11x + 15 = 48$

$11x = 48 - 15$

$11x = 33$

$x = 33 \div 11$

$x = 3$

Check: $11x + 15 \stackrel{?}{=} 48$

$11(3) + 15 \mid 48$

$33 + 15 \mid$

$48 \mid \checkmark$

SELF-CORRECTING EXERCISE B, page 10 (continued).

3. $4x + 1.6 = 6.4$

$4x = 6.4 - 1.6$

$4x = 4.8$

$x = 4.8 \div 4$

$x = 1.2$

Check: $4x + 1.6 \stackrel{?}{=} 6.4$

$4(1.2) + 1.6 \mid 6.4$

$4.8 + 1.6$

6.4 ✓

SELF-CORRECTING EXERCISE C, page 13.

1. $5x + 10x = 45$

$15x = 45$

$x = 45 \div 15$

$x = 3$

Check: $5x + 10x \stackrel{?}{=} 45$

$5(3) + 10(3) \mid 45$

$15 + 30$

45 ✓

2. $7x - 2x = 40$

$5x = 40$

$x = 40 \div 5$

$x = 8$

Check: $7x - 2x \stackrel{?}{=} 40$

$7(8) - 2(8) \mid 40$

$56 - 16$

40 ✓

3. $5.5x - 2.5x = 6$

$3x = 6$

$x = 6 \div 3$

$x = 2$

Check: $5.5x - 2.5x \stackrel{?}{=} 6$

$5.5(2) - 2.5(2) \mid 6$

$11 - 5$

6 ✓

Key to the Exercises in Section 8

Exercise 1

1. (a) +5
   (b) –12
   (c) +100
   (d) –30
   (e) –15

2. (a) +10, +5, 0, –7, –8, –15
   (b) 0, –1, –2, –3, –7, –14

3. (a) +6++3=+9 (b) –6++3=–3 (c) –101+–49=–150

   (d) +82+–143=–61 (e) +152+–77=+75 (f) –79++79=0

Exercise 2

1. $17 + x = 35$
   $x = 35 - 17$
   $x = 18$

2. $-14 + x = -20$
   $x = -20 + +14$
   $x = -6$

3. $x - 13 = 4$
   $x = 4 + 13$
   $x = 17$

4. $1.6 + x = 5.7$
   $x = 5.7 - 1.6$
   $x = 4.1$

Exercise 3

1. $9x = 99$
   $x = 99 \div 9$
   $x = 11$

2. $6 \times x = 96$
   $x = 96 \div 6$
   $x = 16$

3. $7x = 4.9$
   $x = 4.9 \div 7$
   $x = 0.7$

4. $144x = 1728$
   $x = 1728 \div 144$
   $x = 12$

Exercise 4

1. $8x - 9 = 63$
$8x = 63 + 9$
$8x = 72$
$x = 72 \div 8$
$x = 9$

2. $8x + 11 = 27$
$8x = 27 - 11$
$8x = 16$
$x = 16 \div 8$
$x = 2$

3. $14x - 28 = 28$
$14x = 28 + 28$
$14x = 56$
$x = 56 \div 14$
$x = 4$

4. $1.2x + 5 = 125$
$1.2x = 125 - 5$
$1.2x = 120$
$x = 120 \div 1.2$
$x = 100$

Exercise 5

1. $4x + 14x = 54$
$18x = 54$
$x = 54 \div 18$
$x = 3$

2. $15x - x = 42$
$14x = 42$
$x = 42 \div 14$
$x = 3$

3. $2.5x - 1.5x = 7$
$1.0x = 7$
$x = 7$

4. $18x + 9x = 81$
$27x = 81$
$x = 81 \div 27$
$x = 3$

Exercise 6

1. $\frac{x}{25} = \frac{25}{125}$

$125x = 25 \times 25$

$125x = 625$

$x = 625 \div 125$

$x = 5$

2. $\frac{x}{20} = \frac{42}{10}$

$10x = 20 \times 42$

$10x = 840$

$x = 840 \div 10$

$x = 84$

3. $\frac{30}{45} = \frac{x}{3}$

$45x = 3 \times 30$

$45x = 90$

$x = 90 \div 45$

$x = 2$

4. $\frac{6}{28} = \frac{x}{14}$

$28x = 6 \times 14$

$28x = 84$

$x = 84 \div 28$

$x = 3$

Exercise 7

1. $-15 + x = 60$

$x = 60 + 15$

$x = 75$

2. $75x = 900$

$x = 900 \div 75$

$x = 12$

3. $\frac{x}{3} = \frac{36}{12}$

$12x = 3 \times 36$

$12x = 108$

$x = 108 \div 12$

$x = 9$

4. $8x - 72 = 144$

$8x = 144 + 72$

$8x = 216$

$x = 216 \div 8$

$x = 27$

5. $35x + 90x = 1250$

$125x = 1250$

$x = 1250 \div 125$

$x = 10$

6. $5x - 3.2 = 6.8$

$5x = 6.8 + 3.2$

$5x = 10.0$

$x = 10 \div 5$

$x = 2$

M.E.C. - B.N.C.
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